Optimal Vehicle Path Planning in Lane-free Traffic

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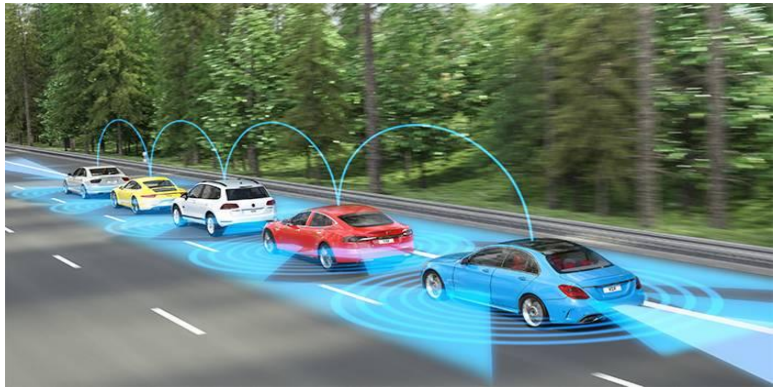
Introduction

➢ Joint Path Planning
  - ❑ A group of CAVs
  - ❑ Lane-free traffic
  - ❑ Vehicle nudging
  - ❑ Open loop optimal control problem
  - ❑ Safe and efficient
  - ❑ Low CPU time
  - ❑ Long horizons
  - ❑ Dozens of vehicles
  - ❑ Many applications

[1] N. Dabestani, P. Typaldos, V. K. Yanumula, I. Papamichail and M. Papageorgiou, "Joint trajectory optimization for multiple automated vehicles in lane-free traffic with vehicle nudging", *Conference on Intelligent Transportation Systems (ITSC) Proceedings*, pp. 5954-5961, 2023.
[2] N. Dabestani, P. Typaldos, V. K. Yanumula, I. Papamichail and M. Papageorgiou, "Joint path planning for multiple automated vehicles in lane-free traffic with vehicle nudging," in *IEEE Transactions on Intelligent Transportation Systems*, vol. 25, no. 11, pp. 18525-18536, 2024.

Introduction

- Vehicle platooning
  - Vastly investigated
  - Traffic safety
  - Roadway capacity
  - Fuel consumption
  - Lane-based
- Vehicle platoons in lane-free traffic
  - Joint optimization
  - Optimal Control
  - 1-D
  - Flexible/Snake-like
  - Interruptible



[3] N. Dabestani, P. Typaldos, I. Papamichail and M. Papageorgiou, "Joint trajectory planning for 1-D snake-like interruptible vehicle platoons in lane-free traffic, " *hEART2024*, Espoo, Finland, June 2024.

Introduction

- Vehicle flocking
  - Inspired by nature
  - Birds, fish,…
  - Largely unexplored
  - Road capacity
  - Energy Saving
  - Reduce the aerodynamic drag force
- Vehicle flocks in lane-free traffic
  - Joint optimization
  - Optimal Control
  - 2-D
  - Any shape
  - Deformable

[4] N. Dabestani, P. Typaldos, I. Papamichail and M. Papageorgiou, "Joint vehicle path planning for interruptible 1-D snake-like platoons and 2D flocks in lane-free traffic," in *IFAC-PapersOnLine*, vol. 58, no. 10, pp.61-68, 2024.

Introduction

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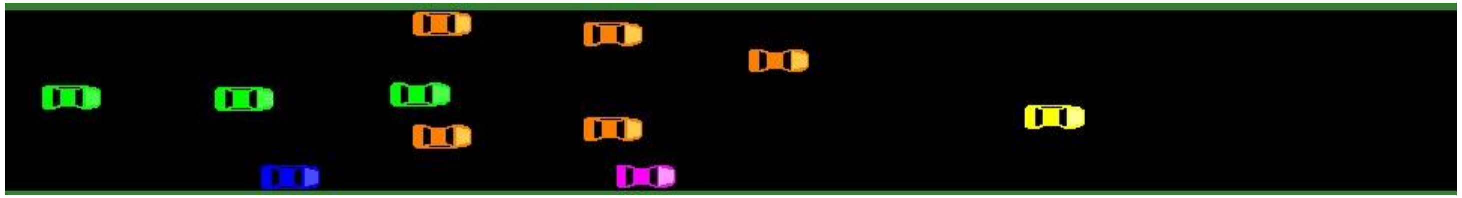
- Open loop OCP ⟹ Distributed Model Predictive Control of CAV Entities
  - Multiple CAV entities
  - Independently
  - Joint OCP
  - Single vehicle OCP
  - Within an event-triggered MPC framework
  - Feasible Direction Algorithm (FDA)
  - Real time solutions



[5] N. Dabestani, D. Troullinos, I. Papamichail, M. Naderi, and M. Papageorgiou, "Distributed Model Predictive Control of Automated Vehicles, Platoons and Flocks in Lane-Free Traffic," in *European Control Conference (ECC25)*, 2025, accepted.

Optimal Control Problem

- **State Equations:**

$$x_i(k+1) = x_i(k) + Tv_{x,i}(k) + \tfrac{1}{2}T^2 a_{x,i}(k)$$
$$y_i(k+1) = y_i(k) + Tv_{y,i}(k) + \tfrac{1}{2}T^2 a_{y,i}(k)$$
$$v_{x,i}(k+1) = v_{x,i}(k) + Ta_{x,i}(k)$$
$$v_{y,i}(k+1) = v_{y,i}(k) + Ta_{y,i}(k)$$

- **Constraints**

- **Longitudinal Acceleration Bounds:** $a_{x,i}^{\min}(\mathbf{x}_i(k)) \le a_{x,i}(k) \le a_{x,i}^{\max}$
- **Lateral Acceleration Bounds:** $a_{y,i}^{\min}(\mathbf{x}_i(k)) \le a_{y,i}(k) \le a_{y,i}^{\max}(\mathbf{x}_i(k))$

Optimal Control Problem

- **Longitudinal Acceleration Bounds**
  - ✓ **Vehicle's acceleration/deceleration characteristics**
  - ✓ **Non-negativity of the longitudinal speed**

$$\max\{-\frac{1}{T}v_{x,i}(k), A_i^{\min}\} \leq a_{x,i}(\boldsymbol{x}_i(k)) \leq A_i^{\max}$$

- **Lateral Acceleration Bounds**
  - ✓ **Lateral road boundary**

$$-K_{\text{lat}}\left[y_i(k)-\tilde{y}_i\right]+(\frac{T}{2}K_{\text{lat}}-2\sqrt{K_{\text{lat}}})v_{y,i}(k) \leq a_{y,i}(\boldsymbol{x}_i(k)) \leq -K_{\text{lat}}\left[y_i(k)-\hat{y}_i\right]+(\frac{T}{2}K_{\text{lat}}-2\sqrt{K_{\text{lat}}})v_{y,i}(k)$$

- **State-Dependent Bounds ⟹ Constant Bounds**

$$a_{x,i}(k)=(1-u_x(k))a_{x,i}^{\min}(\boldsymbol{x}_i(k))+u_x(k)a_{x,i}^{\max}$$

$$a_{y,i}(k)=(1-u_y(k))a_{y,i}^{\min}(\boldsymbol{x}_i(k))+u_y(k)a_{y,i}^{\max}(\boldsymbol{x}_i(k))$$

Optimal Control Problem

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➢Objective Function:

Fuel Consumption | Passenger Comfort | Desired Speeds | Decoupling of Speeds | Collision Avoidance Among groups of CAVs | Collision Avoidance Among CAVs in the group

$$J = \sum_{k=0}^{K-1} \{ \sum_{i=1}^{n} [\frac{1}{2} b_1 a_{x,i}^2 + \frac{1}{2} b_2 a_{y,i}^2 + \frac{1}{2} b_3 f_i^{\mathrm{vdx}}(\boldsymbol{x}_i) + \frac{1}{2} b_4 (v_{y,i} - v^*_y)^2 + \frac{1}{2} b_5 f_i^{c}(\boldsymbol{x}_i) + b_6 \sum_{o \in O_i} c_{io}(\boldsymbol{x}_i, \boldsymbol{x}_o) + b_7 \sum_{j=i+1}^{n} c_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j))$$

$$+ \frac{1}{2} \sum_{i=1}^{n-1} [b_8 E(\Delta \boldsymbol{x}_{i+1,i})) + b_9 f_{\mathrm{l}}(\boldsymbol{x}_i, \boldsymbol{x}_l) + b_{10} f_{\mathrm{u}}(\boldsymbol{x}_i, \boldsymbol{x}_l) + b_{11} f_{\mathrm{b}}(\boldsymbol{x}_i, \boldsymbol{x}_l))]\} + b_{12} \sum_{i=1}^{n} \left[a_{x,i}(0) - a_{x,i}^{\mathrm{prev}} \right]^2$$

Platooning | Flocking | Deviation of Longitudinal Acceleration Between Planning Horizons

Optimal Control Problem

- **Platooning:**

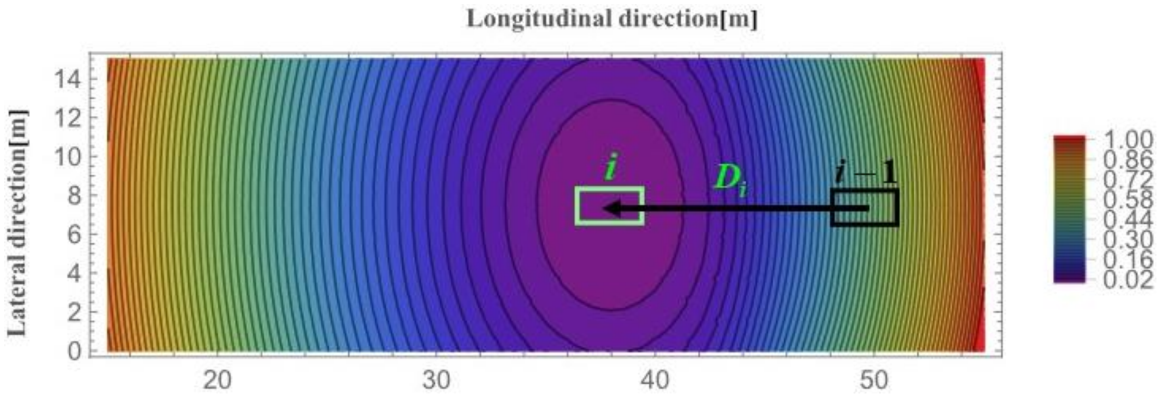
$$(x_i, y_i) \Rightarrow (x_{i-1} - D_i, y_{i-1})$$

Euclidean Distance $\Rightarrow (\Delta x_{i-1,i} - D_i)^2 + \eta \Delta y_{i-1,i}^2$

A nonlinear function of Euclidean Distance $\Rightarrow E(\Delta \boldsymbol{x}_{i-1,i})$

$$E(\Delta \boldsymbol{x}_{i-1,i}) = \left[\frac{1-c}{1+e^{-\gamma\left(\Delta x_{i-1,i} - D_i/2\right)}} + c\right]\left[\left(\Delta x_{i-1,i} - D_i\right)^2 + \eta \Delta y_{i-1,i}^2\right]$$

$$D_i = D_0 + \omega_{\text{at}} v_{x,i}$$



Optimal Control Problem

- **Flocking Application:**

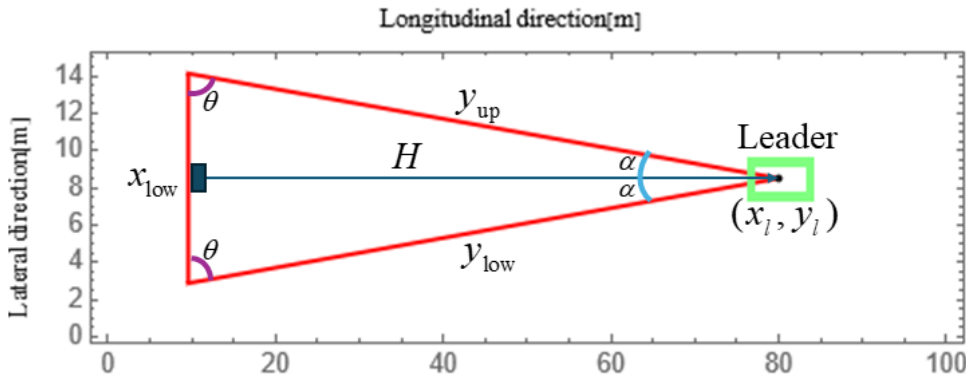
$$y_{\text{up}} = y_l - s(x_i - x_l)$$
$$y_{\text{low}} = y_l + s(x_i - x_l)$$
$$x_{\text{low}} = x_l - H$$

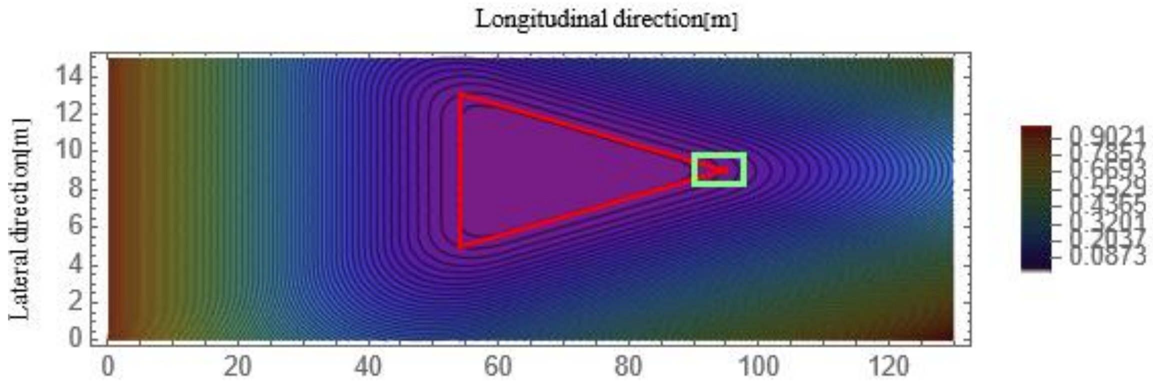
$$H = \left\lceil \frac{n}{2} \right\rceil \left(L_0 + \omega_{\text{fl}} v_{x,l} \right)$$

$$f_{\text{l}}(\boldsymbol{x}_i, \boldsymbol{x}_l) = \begin{cases} (y_{\text{low}} - y_i)^2 & \text{if } y_i \le y_{\text{low}} \\ 0 & \text{otherwise} \end{cases}$$

$$f_{\text{u}}(\boldsymbol{x}_i, \boldsymbol{x}_l) = \begin{cases} (y_i - y_{\text{up}})^2 & \text{if } y_i \ge y_{\text{up}} \\ 0 & \text{otherwise} \end{cases}$$

$$f_{\text{b}}(\boldsymbol{x}_i, \boldsymbol{x}_l) = \begin{cases} (x_{\text{low}} - x_i)^2 & \text{if } x_i \le x_{\text{low}} \\ 0 & \text{otherwise} \end{cases}$$





Optimal Control Problem

- **Collision Avoidance:**

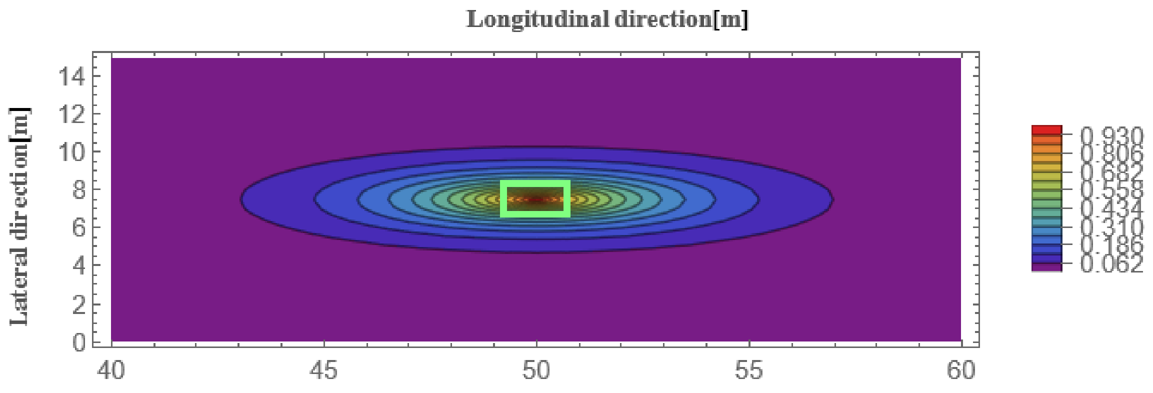
$$c_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j) = \exp^{-d_{ij}}$$

$$L_{ij}^{\mathrm{eff}} = L_{ij} + 0.5(\omega_x(v_{x,i} + v_{x,j}))$$

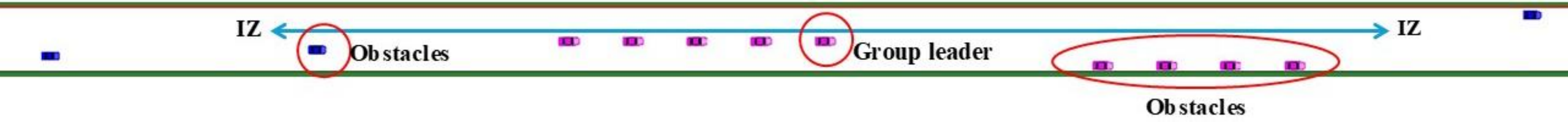
$$W_{ij}^{\mathrm{eff}} = W_{ij} + 0.5\Big(\omega_y\Big((v_{y,i} - v_{y,j})\tanh(y_j - y_i) + \sqrt{(v_{y,i} - v_{y,j})^2 \tanh(y_j - y_i)^2 + \varepsilon_\omega}\Big)\Big)$$

$$p_{ij} = \left(\frac{L_{ij}^{\mathrm{eff}}}{W_{ij}^{\mathrm{eff}}}\right)^2$$

$$d_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j) = v\frac{\sqrt{\left(x_i - (x_j - S_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j))\right)^2 + p_{ij}(y_i - y_j)^2}}{L_{ij}^{\mathrm{eff}}}$$



Model Predictive Control Scheme (MPC)



- The first vehicle in the group ⇨ The group leader
- Interaction Zone (IZ) ⇨ An area upstream and downstream of the group leader
- IZ area ⇨ Leader’s desired speed * the planning horizon of (K)s
- Any non-group vehicle in the IZ ⇨ Obstacle
- Joint / single-vehicle OCP for (K)s

Model Predictive Control (MPC)

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- The OCP is re-solved online, if:

  a) The group has executed the portion of its previous trajectory corresponding to the application period

  b) A new obstacle enters the IZ

  c) A substantial deviation is detected between the predicted and actual trajectories of any surrounding dynamic obstacle

Numerical Solution

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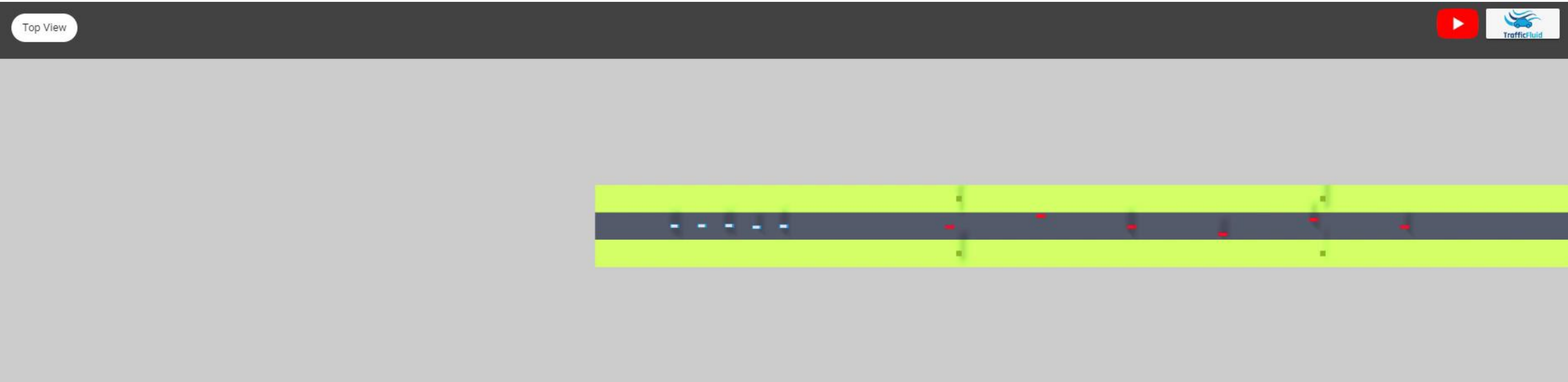
- A Feasible Direction Algorithm (FDA) is employed to solve the Optimal Control Problem in an MPC framework

- FDA is an iterative method that starts with a feasible initial guess for control trajectories

- At each iteration, FDA tries to find a new control vector that corresponds to a local minimum of the cost function

Simulation Results for Scenario 1

➢ **Scenario 1 - Open loop : (Simulation time horizon = 150 s)**

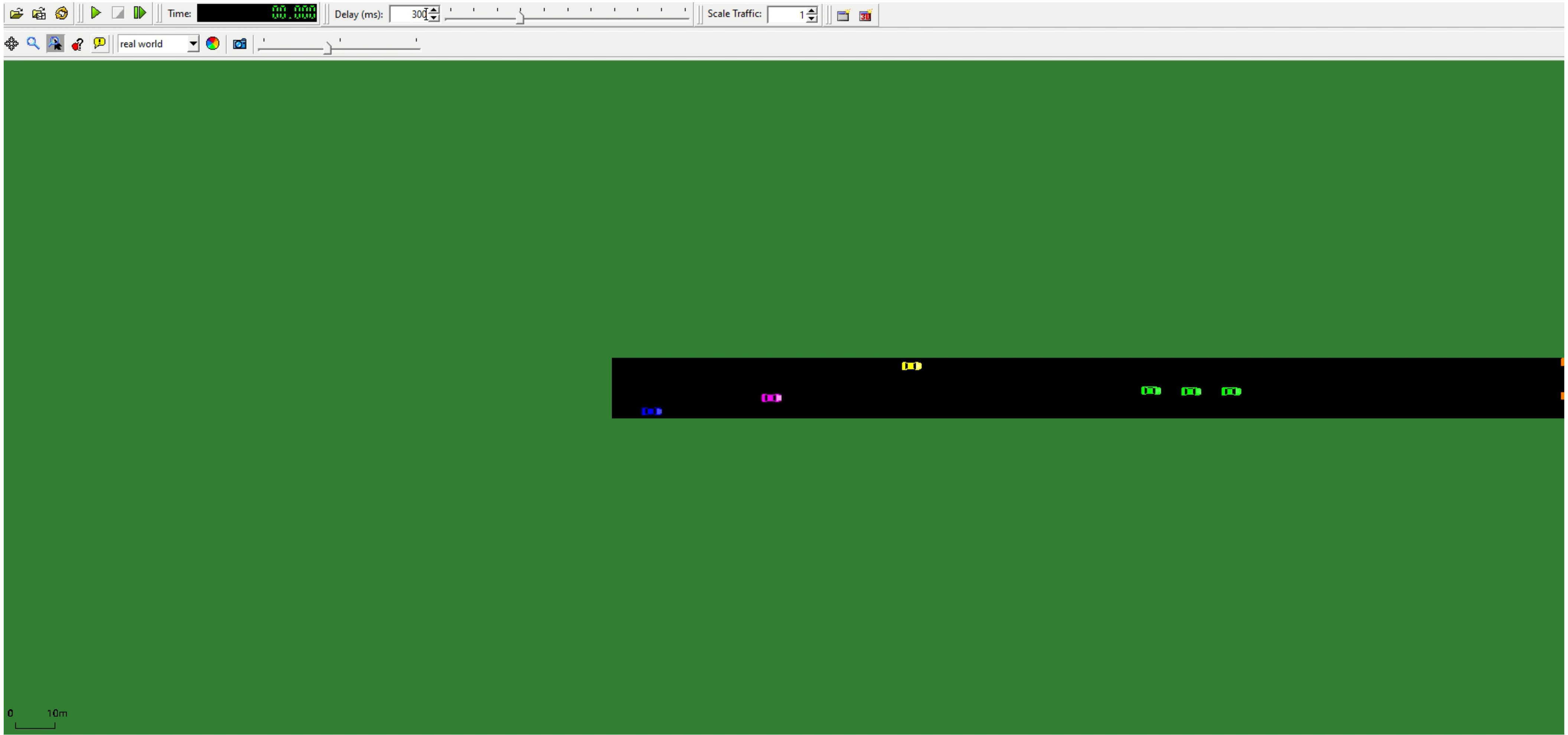


Simulation Results for Scenario 2

- **Scenario 2 – Open loop : (Simulation time horizon = 150 s)**

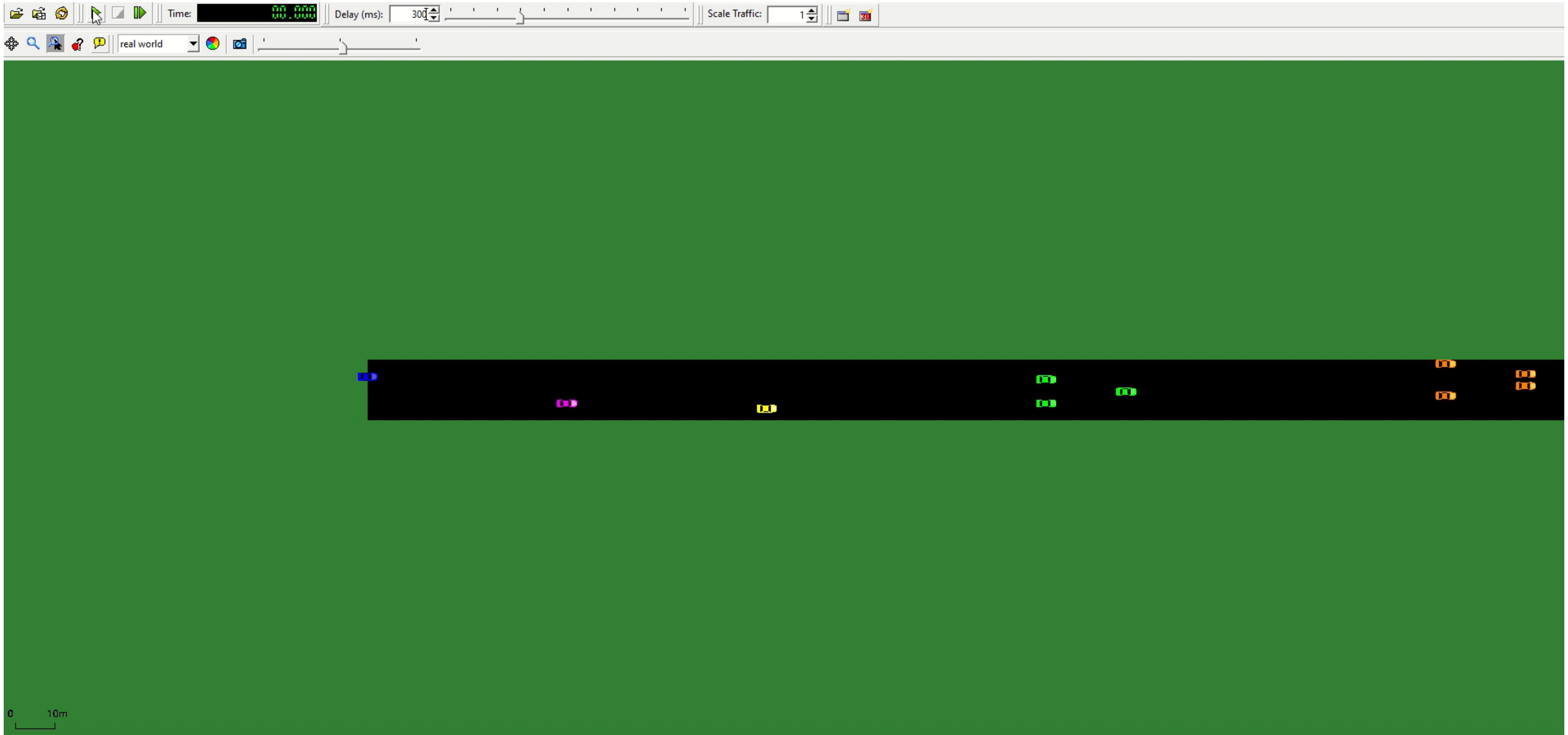
Simulation Results for Scenario 3

➢Scenario 3 – MPC : (Simulation time horizon = 140 s)



Simulation Results for Scenario 4

➢ **Scenario 4 – MPC : (Simulation time horizon = 100 s)**



Future Works

- Inclusion of on-ramps/off-ramps
- Creating weaving section and jointly optimizing the whole network
- Creating congestion and observing the algorithm behavior

Thank you for your attention!

https://www.trafficfluid.tuc.gr

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