Young Researchers Seminar 2013

Lyon, France , June 5-7 2013

Laboratoire d'Economie des Transports
Unité Mixte de Recherche du CNRS n° 5593
Université Lumière Lyon 2 - ENTPE



# Stochastic Shortest Path Problem with Coordinated Traffic Signals

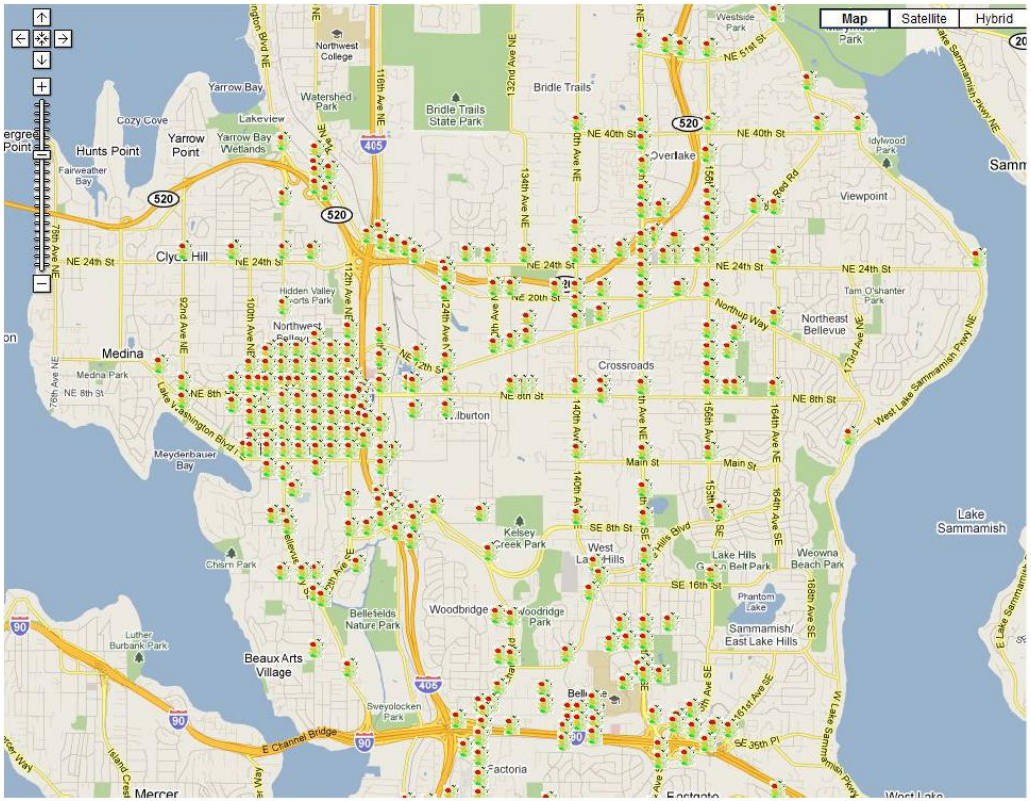
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University of Minnesota, USA

ECTRI
European Conference of Transport Research Institutes

FERSI
Road Safety Research

# Traffic signals on urban arterial

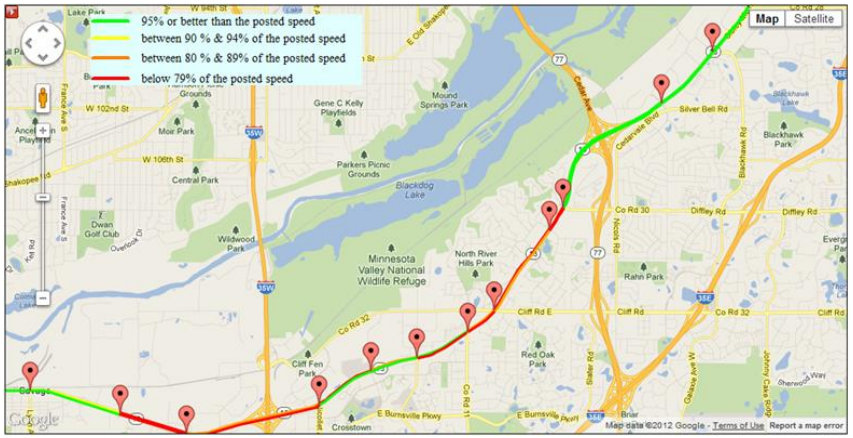



Soruce: Wu et al. 2011

# Traffic signal applications




Green Driver (apple app store)




SMART-Signal system
http://dotapp7.dot.state.mn.us/smartsignal/

# Shortest path problem with traffic signals

- Chen & Yang (2000).
  - Static link travel time.
  - Fixed timing control.
- Ahuja et al. (2002).
  - Dynamic link travel time.
  - Fixed timing control.
- Yang & Miller-Hooks (2004).
  - Stochastic link travel time.
  - Adaptive control.

# Markov decision process (MDP)

- States: $S$.
- Actions: $A_i, i \in S$.
- Costs: $c\big(i, a(i)\big), i \in S, a(i) \in A_i$.
- Transition probabilities:

$$P(j|i, a(i)), j \in S, i \in S.$$

- Goal: minimize expected total cost.

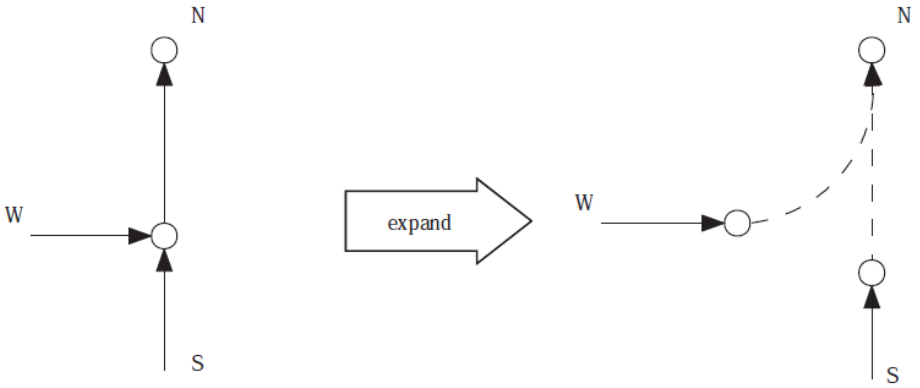
# State space

$$S_v = \{(v,t): v \in V_n, n \in N, t \in \{t_1, t_2, \dots, t_{\gamma(v)}\}\}$$

$N$: set of intersections in the traffic network.

$V_n$: set of arrival nodes extended from $n$.

$t$: possible arrival times.



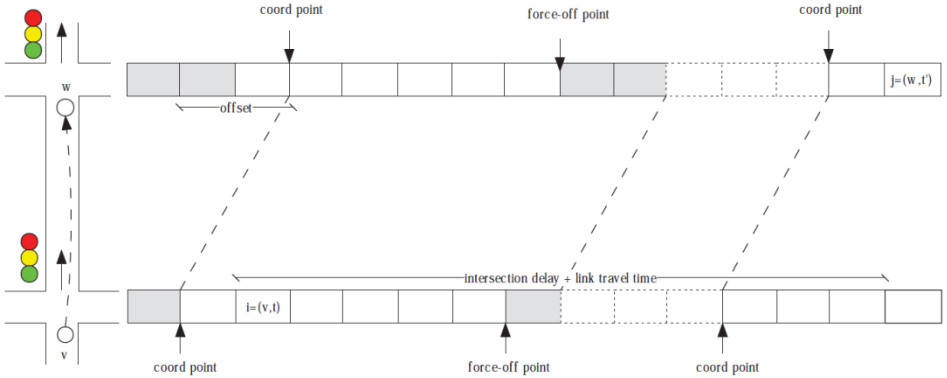
# Action and cost

- Action: $a(i) \in A_i \subset V_N$ is a downstream node of $v$.
- Cost: $E(\delta_r + \delta_q + \tau | i, a)$
  - $\delta_r$:red light delay; $\delta_q$: queuing delay; $\tau$: link travel time.

# Transition probability

$$P(j|i,a) = P(t' = \left(t + \delta_r + \delta_q + \tau - \xi\right)\%\gamma | i, a)$$

- $\delta_r$:red light delay; $\delta_q$: queuing delay; $\tau$: link travel time.
- $\xi$ is the offset between these two coordinated. intersections.
- $\gamma$ is the cycle length at $w$. % is the modulo operation.

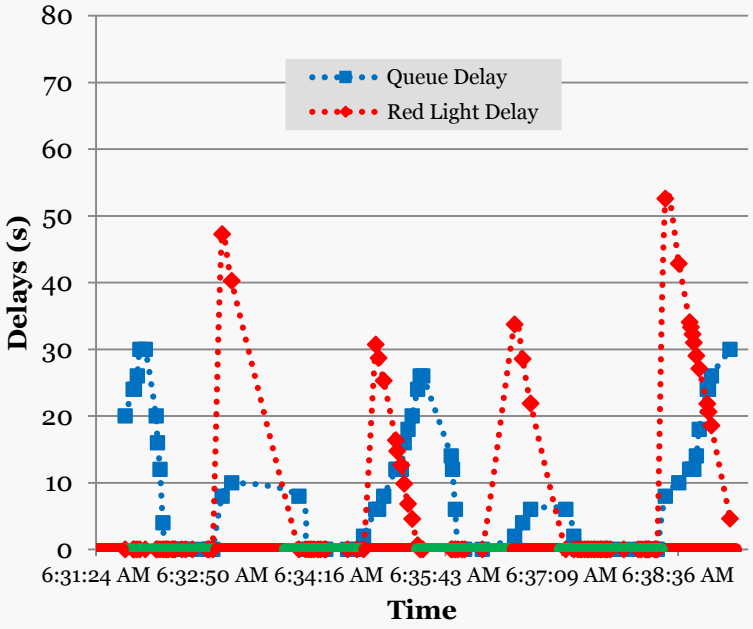


# Objective function

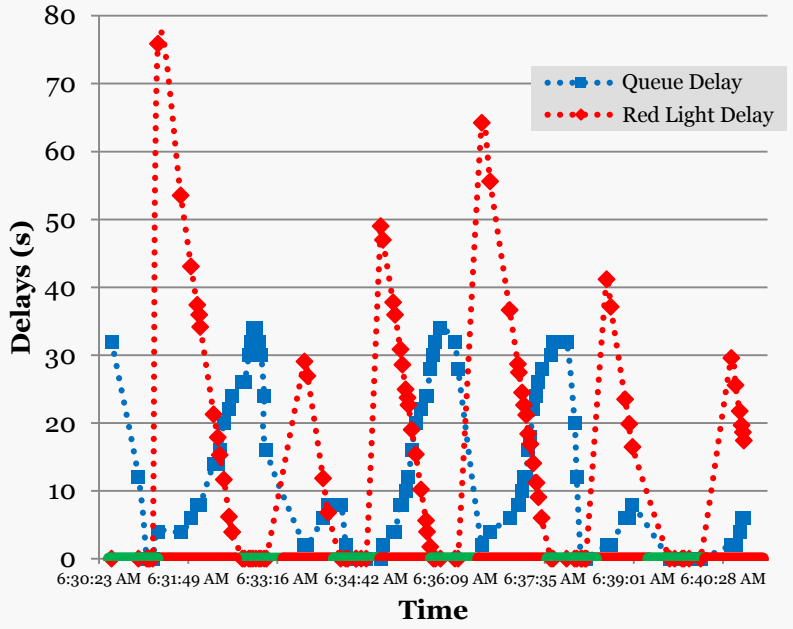
*Minimize*

$$u_\pi(s) = \lim_{X \to \infty} E\{\sum_{k=0}^{X-1} c(i_k, a_k) | i_0 = s\}.$$

# Cost calculation

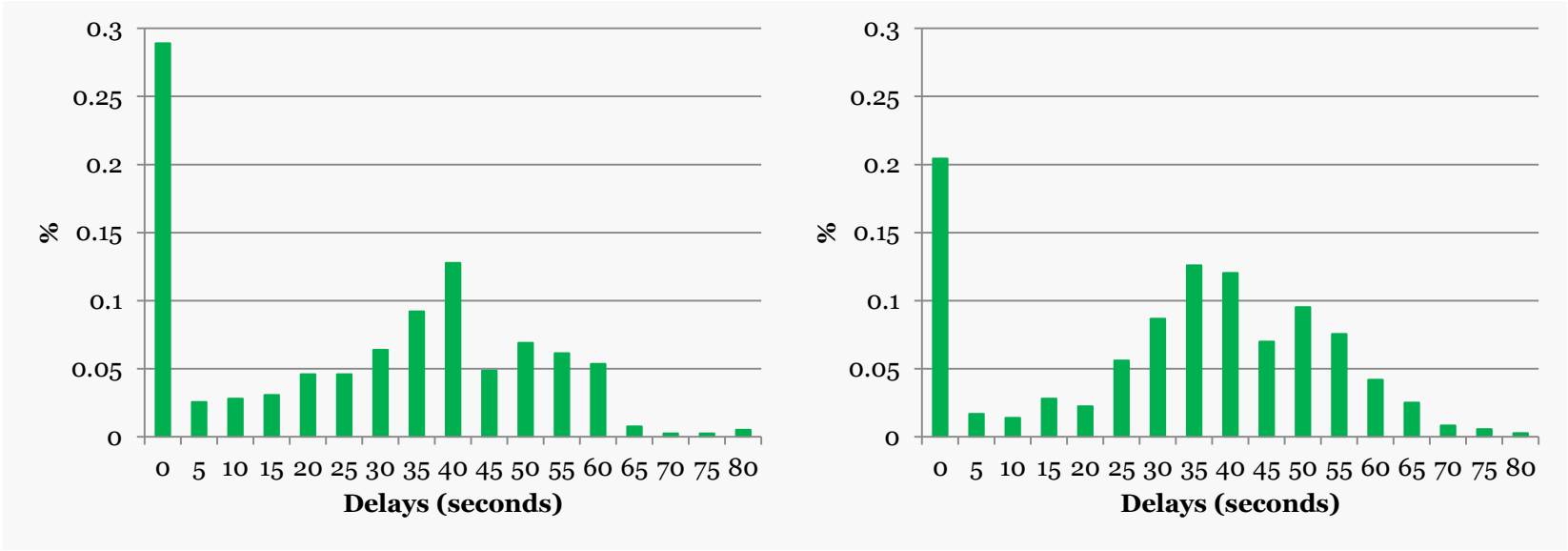



Delays, Sep-09-2009




Delays, Sep-10-2009

# Intersection delay histogram



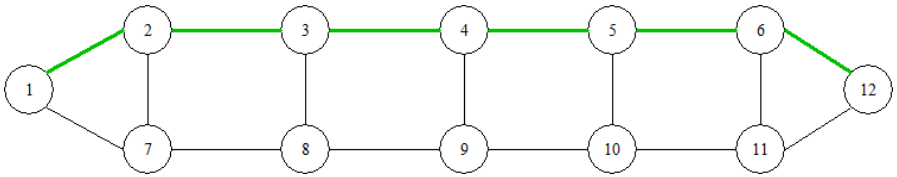

Distr, Sep-09-2009

Distr, Sep-10-2009

# Value iteration algorithm

**Data**: traffic network, signal settings, intersection delay distributions, link travel time distributions, destination
**Result**: expected time cost at each state, the navigation policy to the destination with minimal expected time cost
**for** *each state* $i \notin$ *goal state* **do**
  $u_0(i) = \infty$;
**end**
**for** *each state* $i \in$ *goal state* **do**
  $u_0(i) = 0$;
**end**
**while** $\forall i, \parallel u_k(i) - u_{k-1}(i) \parallel < \varepsilon$ **do**
  **for** *each state* $i$ **do**
    **for** *each action* $a$ **do**
      compute $Q_k(i,a) = c(i,a) + \sum_j P(j|i,a)u_{k-1}(j)$;
    **end**
    compute and store $\pi_k^*(i) = \underset{a}{\operatorname{argmin}}\, Q_k(i,a)$;
    compute and store $u_k(i) = Q_k(i, \pi_k^*(i))$;
  **end**
**end**
**return** $\pi_k^*(i), u_k(i)$

# Preliminary results – setting




network

- Cycle length: 4 unit time.
- Offset: 2 unit time.

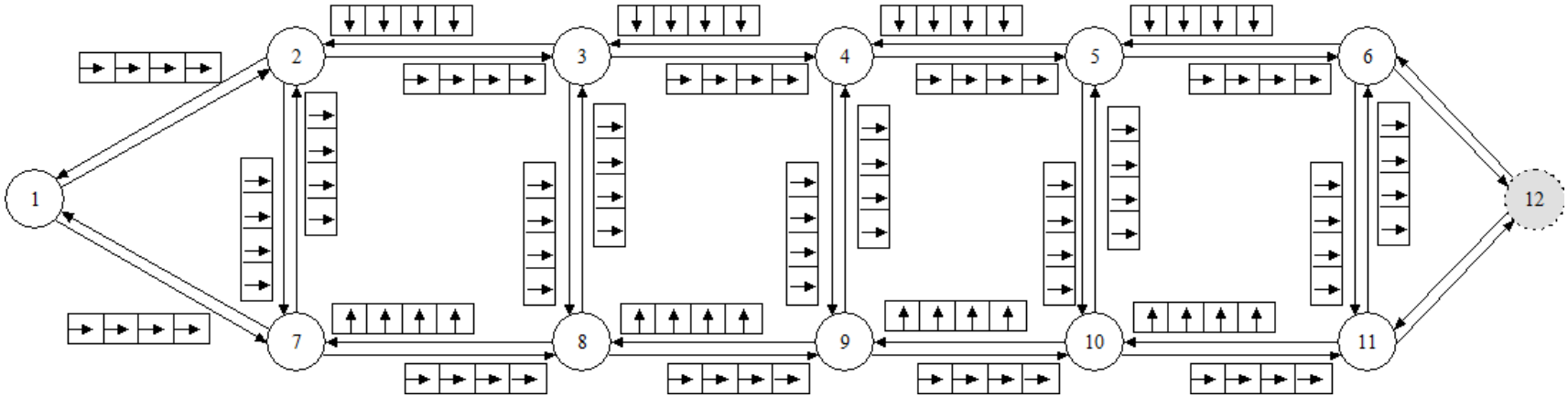
| delay (unit time) | 1 | 2 | 3 |
|---|---|---|---|
| probability | 0.1 | 0.8 | 0.1 |

Link travel time distribution

| | delay | | | | probability | | | |
|---|---|---|---|---|---|---|---|---|
| cycle time | 0 | 1 | 2 | 3 | 0 | 1 | 2 | 3 |
| right | 0,1 | 0,1 | 0,1 | 0,1 | 0.9,0.1 | 0.8,0.2 | 0.4,0.6 | 0.2,0.8 |
| left | 2,1 | 2,1 | 2,1 | 2,1 | 0.2,0.8 | 0.2,0.8 | 0.2,0.8 | 0.2,0.8 |
| straight | 0,1,2 | 0,1 | 0,1 | 0 | 0.1,0.8,0.1 | 0.3,0.7 | 0.8,0.2 | 1 |

Intersection delay distributions

# Preliminary results - test 1



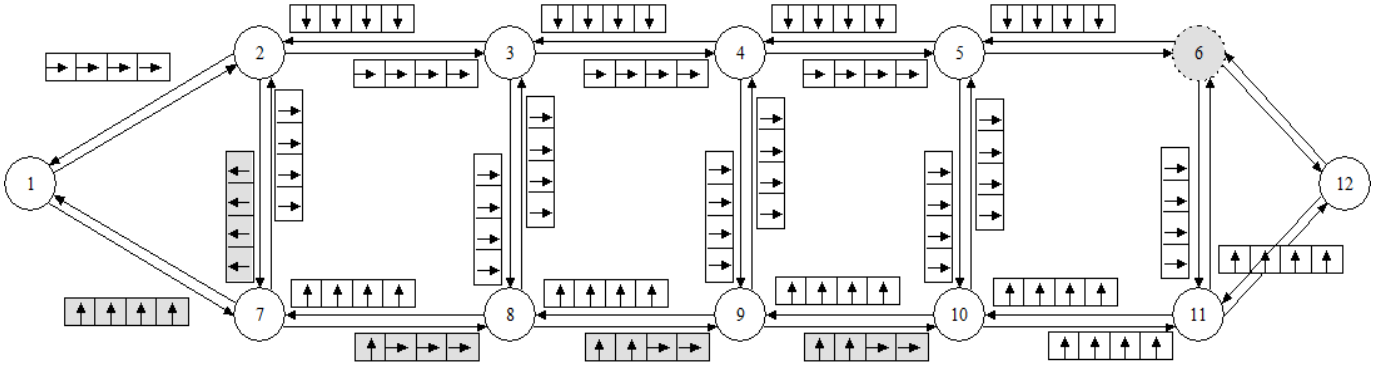

| | Expected path time cost | | | |
|---|---|---|---|---|
| route | Time 1 | Time 2 | Time 3 | Time 4 |
| 1-2-3-4-5-6-12 | 14.758 | 13.989 | 13.250 | 12.933 |
| 1-7-8-9-10-11-12 | 15.375 | 15.075 | 14.575 | 14.375 |

# Preliminary results - test 1

| Offset = 2 | Expected path time cost | | | |
|---|---|---|---|---|
| route | Time 1 | Time 2 | Time 3 | Time 4 |
| 1-2-3-4-5-6-12 | 14.758 | 13.989 | 13.250 | 12.933 |
| 1-7-8-9-10-11-12 | 15.375 | 15.075 | 14.575 | 14.375 |

| Offset = 1 | Expected path time cost | | | |
|---|---|---|---|---|
| route | Time 1 | Time 2 | Time 3 | Time 4 |
| 1-2-3-4-5-6-12 | 15.018 | 14.787 | 14.410 | 14.504 |
| 1-7-8-9-10-11-12 | 15.375 | 15.075 | 14.575 | 14.375 |

# Preliminary results - test 2




| | Expected path time cost | | | |
|---|---|---|---|---|
| route | Time 1 | Time 2 | Time 3 | Time 4 |
| 1-2-3-4-5-6 | 12.540 | 11.775 | 11.038 | 10.719 |
| Node 7 to node 6 | 12.885 | 12.885 | 12.481 | 12.281 |

# Future researches

- State space reduction
- Other solution methods
- Constrained shortest path problem

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# Thank you!

Questions or suggestions?