Young Researchers Seminar 2013

Lyon, France , June 5-7 2013

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# Investigating uncertainty in BPR formula parameters

## The Næstved model case study

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# Outline

- Introduction and rationale
- Case study and methodology
- Results
- Conclusions and perspectives

# Introduction

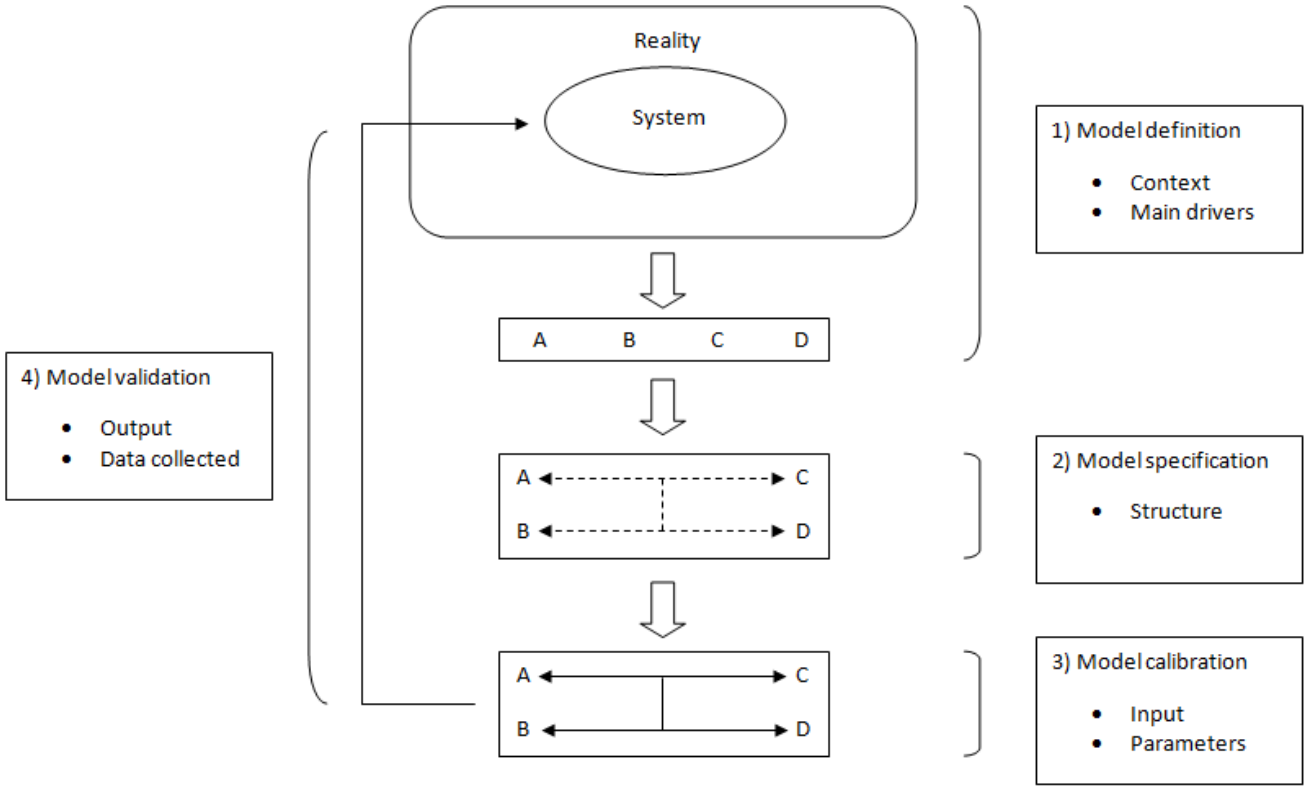
- An extensive literature has demonstrated that there is a considerable and almost systematic inaccuracy between forecasted and observed traffic flows; one of the reasons of this inaccuracy is the complexity of the systems generating demand of transport

- A complex system is a system whose components interact in a way that is difficult to understand, thus making the emerging behaviour (i.e. the system output) difficult to predict. When reproducing complex systems, uncertainty prevents from modelling with a deterministic approach

- Uncertainty:“Any departure from the unachievable ideal of complete deterministic knowledge of the relevant system” (Walker 2003); it refers to limited knowledge (epistemic) or stochasticity (ontological) of some model components and the way they interact

- Transport models reproduce complex systems thus their output becomes unpredictable because of inherent uncertainty

# Introduction



# Introduction

- As a consequence of their inherent uncertainty, transport models “point” output only represents one of the possible output generated by the model

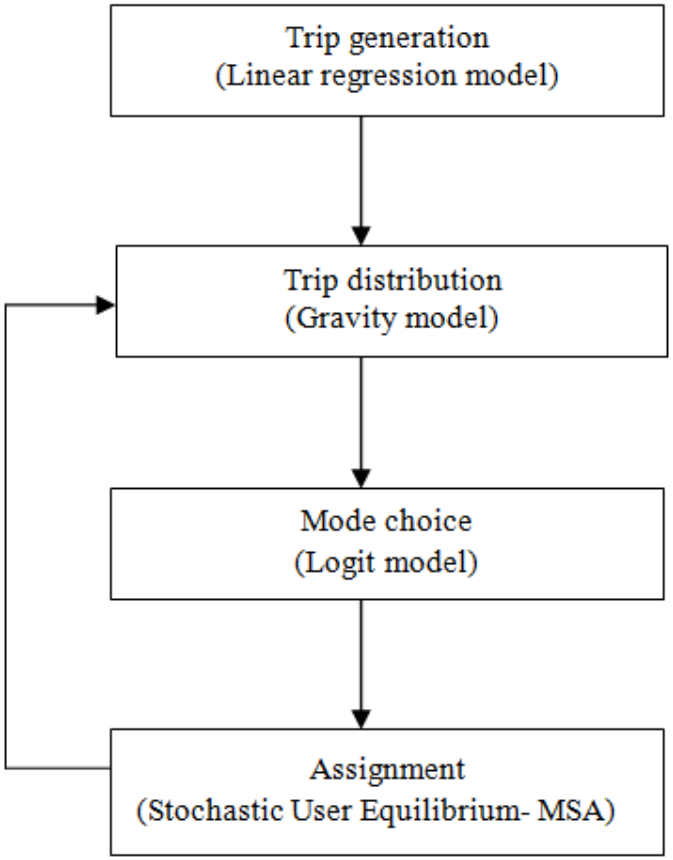
- “Modelled output is better expressed as a central estimate and an overall range of uncertainty margins articulated in terms of values and likelihood of occurrence” (Boyce 1999)

- Uncertainty analysis pertains to
  - quantify uncertainty in (each) model component
  - quantify the overall uncertainty in the model by expressing the model output as a distribution

- The research described in this presentation focused on the effects of uncertainty in the BPR formula parameters on a four-stages transport model (output)

# Case study and methodology

- Næstved model
  - Population: 42,000/ 80,000 (681km$^2$)
  - Trips (24h): 88,500 (10% PT)
  - Low congestion
  - 106 zones, 315 links
  - Traffic is modelled in:
- 2 categories: home/work
  - and business trips
- 2 modes: private and PT
- 24H time interval
  - Four-stage model
    - (3 overall iterations)



# Case study and methodology

- Within traffic assignment models, the BPR formula works as a link performance function; given free flow travel time, (modelled) traffic flow and link capacity, it uses parameters (α, β, γ) to represent different relationships between travel time and traffic flow according to various types of roadways and circumstances.

$$T_r = TF_r * \left\{ 1 + \alpha * \left[ \frac{Flow_r + \gamma\left(Flow_r^{'}\right)}{Capacity_r} \right]^{\beta} \right\}$$

$$V_r = \frac{VF_r}{1 + \alpha * \left[ \frac{Flow_r + \gamma\left(Flow_r^{'}\right)}{Capacity_r} \right]^{\beta}}$$

- This approach has two drawbacks:
  - speed does not precisely reflects travel time
  - BPR function is not able to model speed in congested conditions

# Case study and methodology

- The BPR formula parameters have inherent uncertainty which originates from:
  - the ignorance of the modeller of the true value of the parameters (epistemic uncertainty) and
  - the stochastic behaviour of the (true) parameters itself (ontological uncertainty), which potentially vary by drivers behaviour, time of the day, weather conditions, link characteristics, etc.

- BPR formula parameters uncertainty analysis, two steps:
  1) BPR parameters uncertainty quantification (inherent uncertainty)
  2) Sensitivity test on the Næstved model (propagated uncertainty)

# Case study and methodology

1) BPR parameters uncertainty quantification:
   - BPR parameters calibration: Non-linear regression analyses were implemented using observations from two datasets, namely Mastra and Hastrid (Danish road network). The parameters were estimated for three different road classes: highway, urban roads and local roads
   - BPR parameters distribution: through re-sampling technique “Bootstrap”, parameters were repeatedly calibrated on 999 Bootstrap samples to generate parameter distributions

2) Sensitivity test on the Næstved model:
   - Latin Hypercube Sampling (LHS) procedure was then applied to create parameter vectors of 100 draws each which were used to run sensitivity tests on the Næstved model

# Results

**Bootstrap parameters statistics**

| | Parameter | Estimate | StDev | Min | Max | CV* | K-S |
|---|---|---|---|---|---|---|---|
| Highway | alpha | 0.675 | 0.079 | 0.450 | 0.984 | 0.118 | Lognormal |
| | beta | 5.510 | 0.385 | 4.246 | 6.796 | 0.065 | Normal |
| Urban | alpha | 0.166 | 0.006 | 0.149 | 0.183 | 0.035 | Normal |
| | beta | 0.585 | 0.007 | 0.564 | 0.610 | 0.012 | BetaGeneral |
| | gamma | 0.651 | 0.093 | 0.418 | 0.970 | 0.144 | Lognormal |
| Local | alpha | 0.237 | 0.011 | 0.205 | 0.284 | 0.046 | Normal |
| | beta | 1.261 | 0.015 | 1.212 | 1.311 | 0.012 | InvGauss |
| | gamma | 0.193 | 0.038 | 0.081 | 0.328 | 0.197 | Gamma |

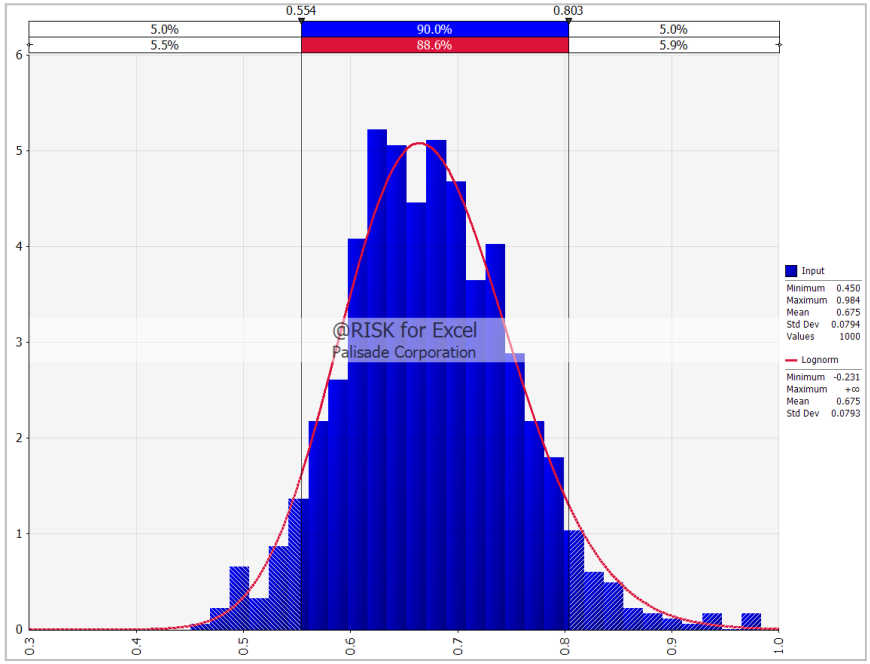
*Coefficient of Variation (CV): StDev/Mean. Commonly used in uncertainty analyses as a measure ofuncertainty

# Results

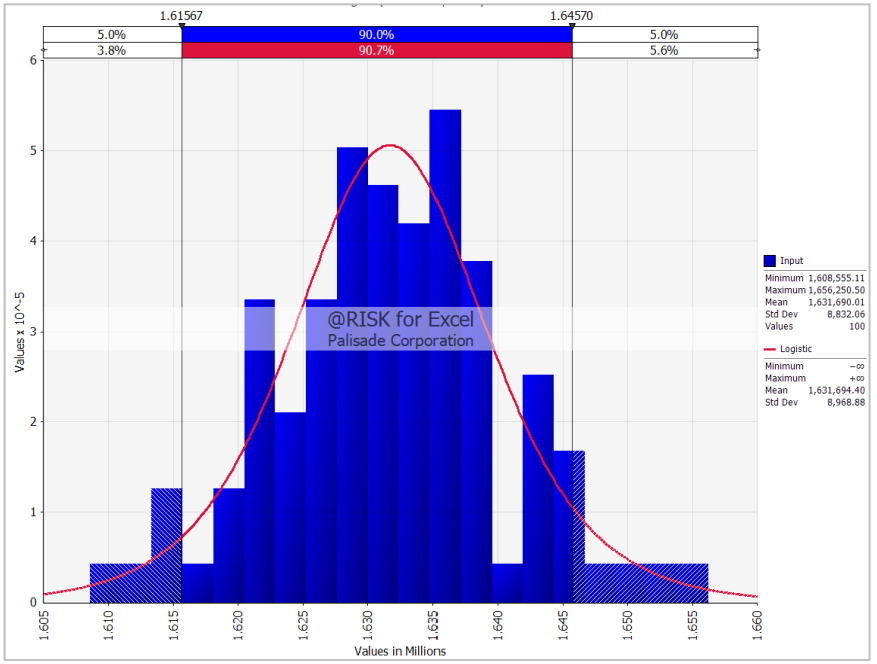
**BPR parameter values comparison**

| | Parameter | Estimate | Kockelman (2001) | Nielsen (2008) | Hansen (2011) |
|---|---|---|---|---|---|
| Highway | alpha | 0.675 | 0.15-4.0 | 0.8-1.2 | 0.5-2.0 |
| | beta | 5.510 | 0.84-5.5 | 1.5-4.0 | 1.4-11 |
| Urban | alpha | 0.166 | 0.15-4.0 | 0.8-1.2 | 0.5-2.0 |
| | beta | 0.585 | 0.84-5.5 | 1.5-4.0 | 1.4-11 |
| | gamma | 0.651 | | | 0.05-2.0 |
| Local | alpha | 0.237 | 0.15-4.0 | 0.8-1.2 | 0.5-2.0 |
| | beta | 1.261 | 0.84-5.5 | 1.5-4.0 | 1.4-11 |
| | gamma | 0.193 | | | 0.05-2.0 |

# Results




Aplha (highway)




Veh-km (highway)

# Results

| Vehicle-Km (links) | |
|---|---|
| | CV |
| Total | 0.127 |
| Highway | 0.040 |
| Urban | 0.249 |
| Local | 0.122 |

- Relevant sensitivity of the model output to the BPR parameters uncertainty, with a CV for all the links of 0.127
- Urban road links show the highest level of uncertainty, followed by local links, probably due to the higher number of route choice alternatives that both networks offer as compared to the highway network

# Results

| Vehicle-Km | | | | |
|---|---|---|---|---|
| | Mean | St Dev | CV | Distribution |
| Total | 2,737,578 | 2,415 | 0.001 | Gamma |
| Highway | 694,335 | 15,320 | 0.022 | Logistic |
| Urban | 411,553 | 11,469 | 0.028 | Loglogistic |
| Local | 1,631,690 | 8,832 | 0.005 | Logistic |

- The uncertainty related to the overall amount of vehicle-kilometre output is small, with a CV of 0.001. This is probably due to the low levels of congestion in the network
- Also in this case different road classes have different sensitivity to BPR parameters uncertainty, with urban roads and highway showing a similar and higher CV as compared to local roads

# Results

| Network travel resistance | | | |
|---|---|---|---|
| | Mean | St Dev | CV |
| Free time | 2,754,855 | 4,391 | 0.001 |
| Cong time | 37,048 | 4,818 | 0.130 |

- The sensitivity tests also demonstrated a relevant sensitivity of the model in terms of modelled congested time whose CV is 0.130

# Conclusions and perspectives

- The results clearly highlight the importance for modelling purposes of taking into account BPR formula parameters uncertainty, expressed as distribution of values, rather than assumed point values. Indeed, the model output demonstrates a high sensitivity to different parameter values and type of distribution

- Different road classes have shown different sensitivity to BPR parameters uncertainty. This seems to suggest the possibility of developing a class reference approach for uncertainty analyses of such kind, so advising further research on the topic

- The analysis produced for the BPR formula parameters different parameter distributions for the three different road classes. These results reaffirm the importance, within sampling procedures, of defining distributions from observed data rather than standard suggested/assumed ones

# Thanks for the attention

Uncertainties in Transport Project Evaluation - UNITE project
(http://www.dtu.dk/subsites/UNITE/English.aspx)

# Extra 1

- When traffic flow reaches the capacity, flow at capacity (FC) and related speed at capacity (SC) the BPR formula curve takes the shape of the dotted curve on the right of FC

- Instead, the observed traffic behaviour is tendentially close to the pattern described by the bold line

- In static assignment models BPR formula is commonly used and accepted for practical reasons, among the others that in this way the speed flow relationship curve is “continuous even beyond capacity and differentiable” (Nielsen and Jørgensen, 2008)

