A stylized PCOD-model for relations between zonal trade flows, transports and distribution centers

Mikal Holmblad

Danish Transport Research Institute, Knuth-Winterfeldts Alle, Build. 116 West, DK-2800 Lyngby, Denmark

Abstract

A stylized freight transport zonal flow model is proposed, concerning the relation between trade flows, transports and the use of distribution centers in a transport system. The model is based on a production/consumption-matrix and an origin/destination-matrix, which represents interrelated but in general non-identical trade and transport flows. The introduction of distribution centers in the model allows transports between these centers to possibly result in decreasing unit transportation costs with increasing transport volume.

Introduction

In many European countries politicians or policymakers have at their disposal a national freight transport model for analyzing impacts of physical and regulatory changes in the national (and international) transport system. Denmark is at present an exception, since the tradition regarding freight modeling has been limited to ad hoc models primarily related to the analysis of specific major infrastructure investments, e.g. Danish bridges like the “Storebælt” link, “Øresundsbron”, or the proposed Femer Belt fixed link.

Recently, a project has been initiated by the Danish Ministry of Transport and the Greater Copenhagen Authority in order to propose a design for a Danish national freight transport model system. The study is carried out by the Danish Transport Research Institute in collaboration with Center for Traffic and Transport, Technical University of Denmark. The project consists of 4 activities: 1) a demand survey on the needs of potential users of a national freight model, 2) a survey and description of currently available data sources and statistics, 3) a study of methods used in freight transport modeling and experiences from other countries, and finally 4) a phase where the results from the previous activities are synthesized in to a proposal for the design of a Danish national freight transport model and a scheme for its realization. Results from these studies are reported elsewhere in e.g. Fosgerau (2003), Fosgerau & Nielsen (2003), and Hansen (2003). The work and the proposed PCOD-model presented in this paper are inspired by the project.

The base data of (national) freight transport models are matrixes describing zonal trade flows between production and final consumption (PC-matrixes) and matrixes describing zonal transports between (apparent) origins and (temporary) destinations (OD-matrixes). Commonly, the PC- and OD-matrixes are taken as approximately the same. For example, in the Swedish national freight transport model SAMGODS (SIKA, 2001), national accounts, which apply to the entire Sweden and has no further geographic detail, is disaggregated with a Furness model to interregional trade flows based on zonal data of sector employment to determine regional production and consumption. An observed regional OD-transport pattern for the base year is in a sense employed as a proxy for the PC-matrix. In contrast, in the Norwegian national freight model NEMO/PINGO (Ivanova et al., 2002 & Vold et al., 2002), trade flows are applied as proxies for transport flows.

Zonal trade flows and zonal transports can however be very different. This is related to the importance of transport logistics. A large portion of a trade flow is not necessarily realized in the transportation system as a direct transport between the location of production and the location of final consumption, but possibly involves consolidation at distribution centers or warehousing in other intermediate zones. The consequence of this is that a zonal transportation pattern (OD-matrix) can be quite different from the corresponding zonal trade pattern (PC-matrix). This is generally not considered in most national freight transport models, however an example is the Dutch national freight transport model SMILE (Tavasszy, 2003). It employs a logistics module that in addition to direct transport enables intermediate transports via one or two distribution centers or warehouses to generate the transport pattern.

Nevertheless, national freight transport models are generally structured in an economic module that determines demand for transport, i.e. trade flows (PC-matrixes), and a transport module that handles the necessary supply of transport that meets this demand. Base year PC-matrixes are very often applied as approximations to base year regional OD-matrixes, in spite of the differences that may occur. Generally, there exists a gap between information concerning trade flows and information concerning transport flows, which has inspired the author of this paper to address the question of PC-matrixes and OD-matrixes. The model presented in the following sections is an attempt to formulate a framework and a connection. The proposed scheme treats indirect transports and transport logistics via distribution centers (DC's).

The PCOD-model

The proposed model concerns the relation between zonal transport flows and trade flows. The assumption is that in addition to direct zonal trade transports also a *system* of handling transportation is available, where increasing transport volume can result in decreasing average unit transportation costs, due to e.g. large-scale operational advantages.

The geographical area of interest is divided into zones that locate both production output (P_r) in zone *r* and final consumption (C_s) in zone *s*. This "world" is in the following referred to as PC-land, or level 1. We use in general the indices r and s, when referring to the trade flow (PC_{rs}) from production zone *r* to consumption zone *s*. The trade flow matrix (PC_{rs}) is taken as exogenously given.

To describe the *transportation system* we define a parallel "world" described by distribution centers (DC's) located in the same geographical "world" as production and final consumption (the zones of the PC-land), i.e. each PC-zone (but not necessarily) has an associated DC. However, in this DC-land (level 2), there is only the possibility for transportation and no production or consumption. In the following we will in general refer to transports (OD_{lm}) between zones in DC-land and PC-land by indices expressing *from l to m*. Transport is not restricted to DC-land, however in PC-land this can only be direct transport (*l*=*r* and *m*=*s*).

We now make the connection between the PC-land and the DC-land by a matrix element, $PCOD^{\omega}_{rslm}$, that describes the amount of transport between production zone *r* and consumption zone *s* (PC_{rs}) that is involved in the total transport OD^{ω}_{lm} from *l* to *m* transportation in either PC-land (ω=11), DC-land (ω =22), or transportations involving the transfer between PC-land and the use of the transportation system and DC's (ω=12, 21). It is emphasized that the $PCOD^{\omega}_{rslm}$-element represents the *transport* from zone *l* to zone *m*, and *not* the detailed choice of route. The model structure and separation of transports in PC-land, to/from, and in DC-land are schematically shown in figure 1.

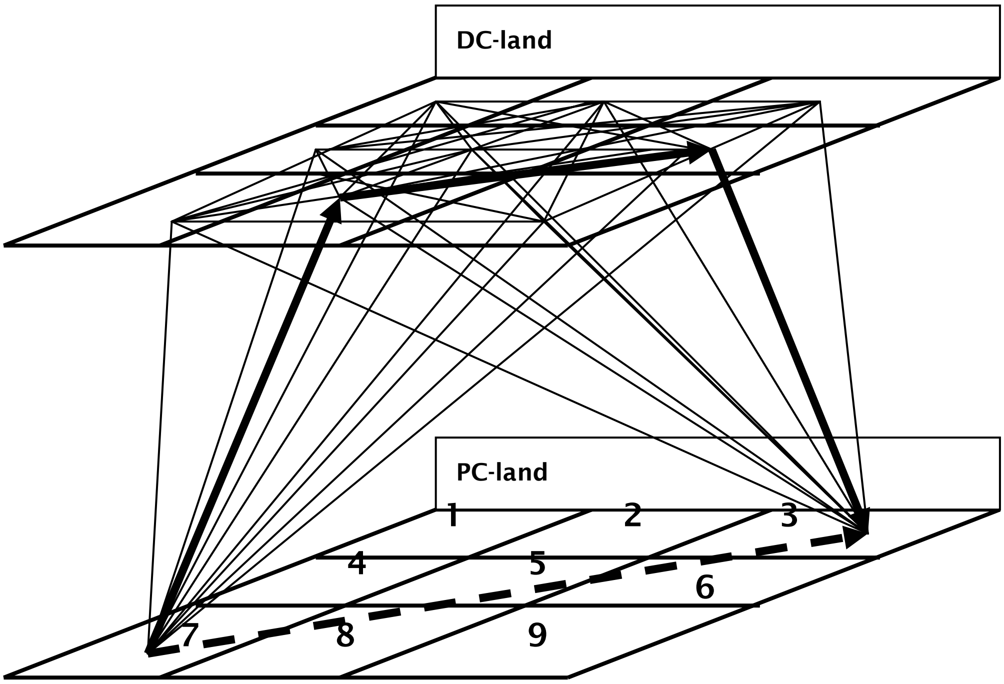

Figure 1: Schematic illustration of PC- and DC-land, showing the direct transportation (dashed arrow), an alternative path involving DC-land (solid thick arrows), and the network of other possible paths.

By this formulation, the exogenously given trade PC_{rs} involves a set of possible transport elements:

(1) $$PC_{rs}:\left\{PCOD^{\omega}_{rslm}\right\} \quad , \quad where \quad PCOD^{\omega}_{rslm}=PC_{rs} \quad or \quad PCOD^{\omega}_{rslm}=0$$

where the elements are of course subject to conditions to be treated later regarding interconnections or transport continuity.

With this formulation it is possible to distinguish between direct (ω=11) and intermediate (ω=12,22,21) OD-transports from l to m:

(2) $$OD_{lm}^{\omega} = \sum_{rs} PCOD_{rslm}^{\omega}$$

and the total (commonly observed) OD-transport for a given *l* to *m* combination is:

(3) $$OD_{lm} = \sum_{\omega} OD_{lm}^{\omega}$$

With the PC-land and DC-land formulation of the problem, zonal materials balances as described below can be deducted:

The production in zone *r* with a fixed *r* to *s* trade (PC_{rs}) is either transported directly from zone *r* to *s* in PC-land, or transported from *r* to any DC-center *k* in DC-land for subsequent transportation:

(4) $$PC_{rs} = PCOD_{rsrs}^{11} + \sum_{k} PCOD_{rsrk}^{12} \quad , \quad \forall r,s$$

Similar, at the consumption zone *s*, the trade (PC_{rs}) is either entering the zone either as a direct transport from zone *r* in PC-land, or as a transport from any center *k* in DC-land:

(5) $$PCOD_{rsrs}^{11} + \sum_{k} PCOD_{rsks}^{21} = PC_{rs} \quad , \quad \forall r,s$$

Finally, a balance for any zone *k* in DC-land for a fixed trade flow *r* to *s* can be formulated as:

(6) $$PCOD_{rsrk}^{12} + \sum_{l} PCOD_{rslk}^{22} = PCOD_{rsks}^{21} + \sum_{m} PCOD_{rskm}^{22} \quad , \quad \forall r,s,k$$

This balance implies that a trade flow from *r* to *s* entering zone *k* in DC-land from either *r* in PC-land or *l* in DC-land can continue either through another DC-destination *m* in DC-land, or return to final consumption in zone *s* in PC-land.

The model as described above is thus straightforwardly formulated as a set of linear equations. However, what is still missing to compute the problem is of course formulating the costs of transportation (and handling at DC's), in order to minimize e.g. system costs.

The system can be kept linear if fixed volume independent costs are employed. All though this is inherently the basis of the model in PC-land, however, the feasible employment of the DC-land depends on the assumption that a transportation *system* can operate more efficiently enabling decreasing unit costs with transport volumes as compared to independent direct from *r* to *s* transports. This was implemented as volume dependent transportation costs in DC-land (TC_{lm}^{22}) modeled by a decreasing function, which was taken as an exponential:

(7) $$TC_{lm}^{22} = TC_{lm}^{11} \cdot \left(A \cdot \exp\left(-\alpha \cdot OD_{lm}^{22}\right) + B \right) \quad , \qquad A + B = 1$$

The transportation costs are thus initially equal to those in PC-land and levels off approaching a fixed fraction (B) of the costs in PC-land. The final system cost to be minimized becomes:

(8) $$SystemCost = \sum_{lm} (OD_{lm}^{11} \cdot TC_{lm}^{11} + OD_{lm}^{12} \cdot TC_{lm}^{12} + OD_{lm}^{21} \cdot TC_{lm}^{21} + OD_{lm}^{22} \cdot TC_{lm}^{22}(OD_{lm}^{22})) \\ + (OD_{lm}^{12} + OD_{lm}^{22}) \cdot DCC_m)$$

In the system cost function, a handling cost at DC's (DCC_m) has additionally been taken in to account for transports involving an end-point DC.

Computational issues

The application of the model is not without complications, and a few computational issues related to non-linear programming (NLP) will briefly be considered in the following.

The non-linear nature and structure of the model result in the presence of several local minima, and makes it non-trivial for a NLP-solver to find acceptable minima. The source of the local minima is the non-linear terms: $OD_{lm}^{22} \cdot TC_{lm}^{22} = OD_{lm}^{22} \cdot TC_{lm}^{11} \cdot (A \cdot \exp(-\alpha \cdot OD_{lm}^{22}) + B)$, i.e. the DC-land transportation costs in the system cost objective function equation 8, which implies that the objective function in combination with the zonal balances equations 4 to 6 becomes non-convex and non-smooth. The aim of this paper is not to address the algorithmic and computational challenges that this implies. It is however important to consider the complication when validating computed solutions. The NLP-solver employed in this work was the CONOPT2-solver, which is part of the GAMS software package. The CONOPT2-solver attempts to locate local solutions to a NLP minimization problem, and is not concerned with the possible existence of a global minimum.

Related to the discussion above, it may be necessary to initialize variables in various ways in order to determine (an) acceptable (local) solution(s) to the NLP minimization problem. One approach applied in the 9 zone test examples is e.g. initially to transport all *r* to *s* trade flows as direct *r* to *s* transport flows in DC-land via the respective local DC's. Other initializations were used as well.

Even with the modest number of 9 zones as employed in the test model described in the previous section, the formulation involves a huge number of variables $PCOD_{rslm}^{\omega}$, and the number of unknowns increases rapidly as $4 \cdot Z^4$ with the number of zones Z. Solutions to the 9 zone test examples are computed in a few minutes on a standard desktop computer, but computation time grows rapidly with an increasing number of zones.

However, quite many of the variables can initially be fixed considering the structure of the model and invoking reasonable restrictions on allowable transports. Secondly, every zone does not necessarily involve a DC.

In the 9 zone test model the following fixed variables were applied:

It was assumed that local production and local consumption generates only local transport in PC-land, which can be formulated by fixed variables as:

$$PCOD_{rslm}^{\omega} = PC_{rs} \;\; for \;\; r = s = l = m \;\; \wedge \;\; \omega = 11, \;\; and \;\; PCOD_{rslm}^{\omega} = 0 \;\; for \; r = s \;\; otherwise$$

Local transports in DC-land were not allowed:

$$PCOD_{rslm}^{\omega} = 0 \ for \ l = m \ \wedge \ \omega = 22$$

Trade from *r* to *s* could not be returned to DC-land at the consumption zone *s*:

$$PCOD_{rslm}^{\omega} = 0 \ for \ l = s \wedge \omega = 12$$

Similar, production in *r* could not be returned from DC-land to the production zone *r*:

$$PCOD_{rslm}^{\omega} = 0 \ for \ m = r \wedge \omega = 21$$

Model tests

The model test cases were computed on synthetic data on trade flows and transportation costs as described below. The 9 zone PC-matrix was kept constant in test cases and is shown in table 1.

PC_{rs}	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	25.0	20.0	30.0	0.0	30.0	30.0	23.0	19.0	34.0
2	0.0	70.0	125.0	0.0	51.0	15.0	98.0	56.0	44.0
3	40.0	0.0	0.0	40.0	0.0	5.0	34.0	35.0	121.0
4	0.0	0.0	10.0	56.0	11.0	123.0	25.0	17.0	0.0
5	20.0	40.0	50.0	72.0	122.0	50.0	137.0	120.0	23.0
6	12.0	26.0	24.0	81.0	8.0	13.0	39.0	15.0	98.0
7	23.0	0.0	34.0	23.0	73.0	34.0	0.0	137.0	56.0
8	123.0	8.0	17.0	0.0	15.0	37.0	34.0	0.0	17.0
9	18.0	34.0	0.0	11.0	4.0	51.0	0.0	24.0	38.0

Table 1. Synthetic trade flows applied in the test cases.

Transport costs between PC-land and DC-land are assumed to be the same as the corresponding zonal transports in PC-land, i.e. $TC^{12}_{lm} = TC^{21}_{lm} = TC^{11}_{lm}$. These artificial zonal transportation costs are kept fixed as shown in table 2.

The various handling costs (DCC_k) at DC's used in the test cases are artificial, as well as the chosen parameters (α and B) applied in the functional relationship between transport costs and volume. They are further assumed the same for all zones.

Table 3 summarizes selected results for some of the test cases. It is emphasized that the shown solutions are not unique since in some cases several solutions with comparable system costs were acceptable. Additionally, the underlying detailed transport pattern for identical system costs can be different related to multiple solutions. Table 3 however illustrates the general behavior of the PCOD-model.

Test case 1 represents a situation where it is generally unfeasible to transport via DC-land, since handling costs at DC's are large, and there is no reduced transportation costs in DC-land. All transports are thus computed to take place as direct transports in PC-land.

In test case 2, the handling costs at DC's are reduced significantly, but still there is no benefit of transporting within DC-land as compared to PC-land, there are thus no transports between DC's in DC-land. However, some transports from a production zone in PC-land can lower transportation costs by transporting to a DC's with a direct return to a consumption zone in PC-land. The "penalty" of a handling cost (DCC_k) at a DC is low enough that combined transportation cost ($TC^{12}_{rk} + TC^{21}_{ks} + DCC_k$) is more feasible than a direct transport exclusively in PC-land. This is also the situation in test case 3, where handling costs at DC's are absent.

TClm	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	0.1	1.0	2.0	1.0	1.5	1.75	2.0	1.75	3.0
2	1.0	0.1	1.2	1.5	1.0	1.5	1.75	2.0	1.75
3	2.0	1.2	0.1	1.75	1.5	1.0	3.0	1.75	2.0
4	1.0	1.5	1.75	0.1	1.0	3.0	1.0	1.5	1.75

5	1.5	1.0	1.5	1.0	0.1	1.0	1.5	1.0	1.5
6	1.75	1.5	1.0	3.0	1.0	0.1	1.75	1.5	1.0
7	2.0	1.75	2.0	1.0	1.5	1.75	0.1	1.0	2.0
8	1.75	2.0	1.35	1.5	1.0	1.5	1.0	0.1	1.0
9	3.0	1.75	2.0	1.75	1.5	1.0	2.0	1.0	0.1

Table 2. Synthetic transportation costs applied in test cases.

In test case 4 to 6, volume dependent transportation costs in DC-land are introduced by setting the parameter $\alpha = 1.0$, and the minimal cost (B) at of half the cost in PC-land for the same zonal relationship. With DC cost as in test case 1, all transport is realized in PC-land (test case 4). In test case 5, a reduced DC cost makes it feasible for some transport to be carried out via DC-land. With further reduction (test case 6), it becomes even more feasible to transport via DC-land and there is essentially only intra zonal production/consumption transport in PC-land. The OD^{22} is larger than $OD^{12} = OD^{21}$, since some transports involve more than 2 DC's in DC-land.

The reduction of transport unit costs in DC-land as applied in test cases 4-6 are quite extreme with $\alpha = 1.0$. The test cases 7 to 10 use more modest values and illustrate how trade flows are transported via PC-land and as direct transport in PC-land depending on the α -parameter defining the unit cost function in DC-land. The minimum cost is fixed at half the cost in PC-land for the same zonal relation, and the handling cost at DC' s are kept fixed at $DCC = 0.1$. In test case 7 and 8, it is generally much cheaper to transport via DC-land, and essentially all transport pass through DC-land. As the feasibility of transporting via DC-land requires larger and larger volumes (test case 9 and 10), the low volume trade flows generally cannot benefit from the transportation system in DC-land, and are thus gradually realized as direct transports in PC-land.

Test case #	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
α	0.0	0.0	0.0	1.0	1.0	1.0	0.1	0.05	0.01	0.007
B	0.0	0.0	0.0	0.5	0.5	0.5	0.5	0.5	0.5	0.5
DCC	2.0	0.1	0.0	2.0	0.5	0.1	0.1	0.1	0.1	0.1

System Cost	4319	4118	4085	4319	4181	3141	3183	3330	3873	4042
Total OD^{11}	2948	2624	2452	2948	2624	324	329	329	687	1054
Total OD^{22}	0	0	0	0	120	2948	2955	3184	2907	2185
Total $OD^{12}=OD^{21}$	0	324	496	0	324	2624	2619	2619	2261	1894

Table 3. Modeling results for test cases.

A more detailed comparison between test cases 8 and 10 illustrates the nature of the model and the possible use of DC's and consolidation of different trade flows in DC-land. Figure 2 shows an example from test case 10, where it is not very attractive to transport via DC-land ($\alpha = 0.007$). The figure illustrates how trade flows from 4 production zones (5, 6, 7, and 9) are transported to the respective consumption zones (1, 4, 5, and 7). Trade flows from zone 7 and 9 to zone 1 consolidate on the transport in DC-land from zone 5 to 1, and trades from zone 5 and 6 to zone 4 consolidate on the transport in DC-land from zone 5 to 4. Remaining trade flows are transported directly in PC-land.

Figure 3 shows how the above situation change as it becomes more feasible to transport via DC-land (test case 8 with $\alpha = 0.1$). All the trades shown in figure 2 from test case 10 become transported via DC-land and significant consolidation of the transports takes place. For example the trades from zone 6 or 9 are initially transported together to the respective local DC's and then to zone 5 in DC-land from where it is separated and distributed to the final consumption zones. Transportation costs are further reduced for the trades from zone 6 and 9 to zone 4, since they are consolidated on the transport from zone 5 to 4. The trade from zone 9 to zone 1 consolidates with trade from zone 5 to zone 1 on the transport between zone 5 and zone 1 in DC-land. Finally, the trade from zone 7 to zone 4 and 1 are transported via the local DC in zone 7 to zone 4, from where the consolidated transport separates to the respective consumption zones.

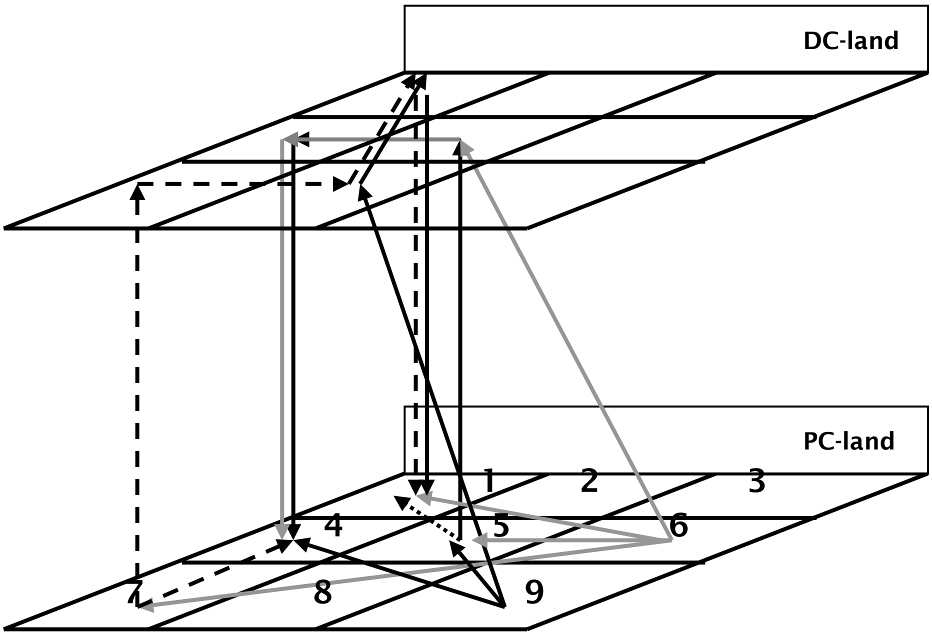

Figure 2. Selected transport pattern involving trades from production zone 5 (dotted line), zone 6 (gray lines), zone 7 (dashed lines), and zone 9 (solid black lines) for test case 10.

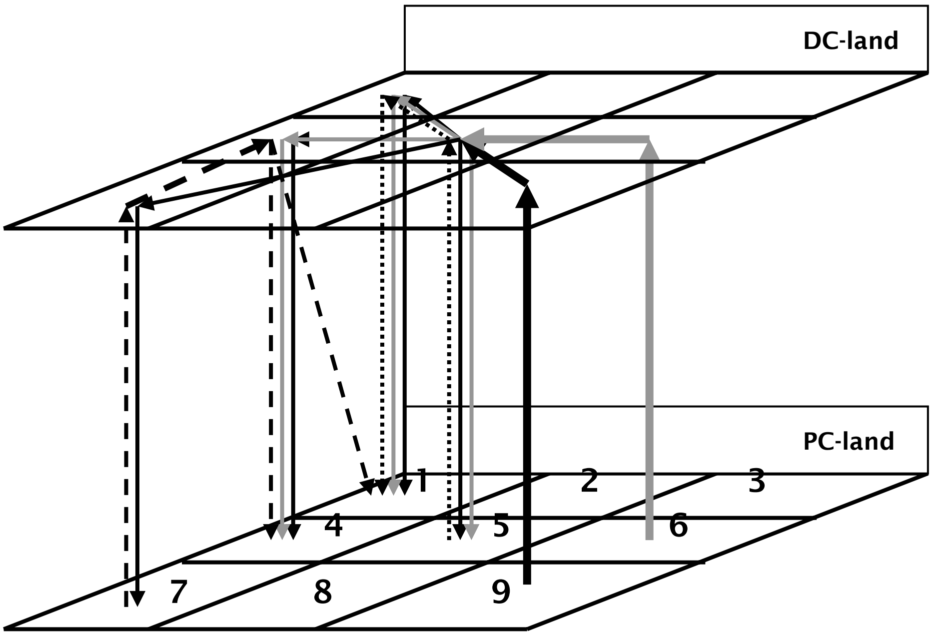

Figure 3. Selected transport pattern involving trades from production zone 5 (dotted line), zone 6 (gray lines), zone 7 (dashed lines), and zone 9 (solid black lines) for test case 8.

Further work

Further development of the model will address the mentioned model inadequacies and computational complications related to multiple solutions and local minima. The non-convexity of the model invokes important and problematic limitations, since even though the system costs of two situations and the total transports within and between PC- and DC-land are almost the same, the detailed zonal transport pattern can however be quite dissimilar. If the system costs of the (local) minima are similar, it means that one solution is as feasible as the other, and the NLP-solver (and the model) can thus in many situations not determine a unique solution (if it exists). However, the problem is expected to become less critical if the zonal transportation cost matrix involves more variation in e.g. transportation costs than the costs used in the shown test cases (table 2). Furthermore, addition of information regarding transports (or traffic), e.g. employment of a few selected "observed" OD-transports, will possibly reduce the number of feasible solutions.

Secondly, PCOD-model trials with realistic inter-zonal trade flows in Denmark will be attempted. For Denmark quite reliable and commodity detailed inter-zonal trade flows are available at the county and municipal level (SAM-K matrixes). The SAM-K's are constructed by the Institute of Local Government Studies (AKF) and the Danish Institute of Agricultural and Fisheries Economics (SJFI) as a spatial differentiation of Danish national input-output tables (Madsen & Jensen-Butler, 1999). Computed OD-results of the PCOD-model will be compared with national statistics on e.g. "National freight transport by Danish trucks" and "Freight transport by rail". Application of the PCOD-model on SAM-K's will furthermore require estimation of reasonable average transportation cost between counties or municipalities.

Finally, the question of commodity classification has not been addressed in this paper. It is however clear that this needs consideration in order to aggregate goods in a way that is compatible with the assumptions of the model transportation *system*. This implies that goods need be aggregated in logistic families showing similar transportation requirements.

Conclusion

A PCOD-model linking intermediate transports (OD-matrixes) via distribution centers (DC's) has been presented to model possible transport patterns resulting from an exogenously given trade pattern (PC-matrix). The non-linear nature of this quite simple model illustrates complications concerning multiple transport pattern solutions, which, however, in turn presumably reflect some aspects the complexity and degrees of freedom in the transport logistics of real-life.

References

Fosgerau, M. (2003). User Demands for a National Freight Transport Model. Conference on National and International Freight Transport Models. Helsinore, Denmark. (http://www.clgdk.dk)

Fosgerau, M. & Nielsen, O. A. (2003). Design of a Danish National Freight Model. Conference on National and International Freight Transport Models. Helsinore, Denmark. (http://www.clgdk.dk)

Hansen, C. O. (2003). Data Issues – A Danish Case. Conference on National and International Freight Transport Models. Helsinore, Denmark. (http://www.clgdk.dk)

Ivanova, O., Vold, A. & Jean-Hansen, V. (2002). PINGO – A model for prediction of regional and interregional freight transport. TØI rapport 578/2002, Norway.

Madsen, B. & Jensen-Butler, C. (1999). Make and Use Approaches to Regional and Interregional Accounts and Models. Economic Systems Research, Vol. 11, No. 3, 1999.

SIKA (2001). The Swedish Model System for Goods Transport – SAMGODS, SAMPLAN rapport 2001:1. (http://www.sika-institute.se)

Tavasszy, I. A., van der Vlist, M. J. M., Ruijgrok, C. J. & Burgess, A (2003). Scenario-wise analysis of transport and logistics systems with a SMILE. TNO Inro, Delft, Netherlands. Presentation at Conference on National and International Freight Transport Models. Helsinore, Denmark (http://www.clgdk.dk)

Vold, A., Andersen, J., Hovi, I. B., Ivanova, O., Jean-Hansen, V., Lervåg, L., Meland, S. & Wahl, R. (2002). NEMO – Nettverksmodell for godstransport innen Norge og mellom Norge og utlandet. TØI rapport 581/2002.