Taxation, Time Allocation and Externalities

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Abstract

The approach introduced by Becker (1965) is used to derive a formula for the optimal tax policy in the case where all factors can be taxed. The same was done in Kleven (2003) concluding that transportation which saves time should carry a lower tax than other types of transport thereby saying that sportscars should be taxed at a lower rate than slower cars. The fact that Kleven does not account for the presence of externalities might be the reason this result emerges so clearly. The present paper derives a formula for the inverse elasticity rule having the result by Kleven as one special case and the additivity property derived by Sandmo (1975) as another special case. That both these results emerge from the same basic model demonstrates the strength of the approach.

1. Introduction

In many cities congestion related problems are increasing. As a result the politicians wish to regulate the traffic and the choice of instruments are vital for the outcome. Should one system be implemented or can other instruments achieve the same at lower costs? What problem is the instrument designed to target? Can a given instrument be used to generate public revenue? Are the chosen instruments politically feasible and how do they interact with the rest of the economy?

Often the concept of marginal cost pricing is mentioned as a way to internalize the external costs of transport and thereby induce an optimal usage of the transport infrastructure. But if a tax instrument is to be used it is important to know how the optimal tax

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scheme is to be designed. It is also important to know how people react to a given tax instrument and how this influence other parts of the economy. Otherwise the outcome and the cost of the instrument might be quite different from what was initially expected.

In the theory of optimal taxation characterisations of the optimal tax scheme has often been derived in different situations. Some of the best known results are the inverse elasticity rule (see for example Ramsey (1927), Sandmo (1976) and Auerbach and Hines (2002)) and the Colett-Hague result (Corlett and Hague (1954)). I will focus on the inverse elasticity rule stating that the tax rate on a given good should be inversely related to the size of its demand elasticity meaning that that goods which are necessities should be taxed more heavily than goods which are not. This is due to the overall goal of reducing distortions induced by the tax system and the question of distributional considerations is therefore ignored. Kleven (2003) demonstrated that the inverse elasticity rule also emerges in the Becker setup though in a modified form.

Another result from tax theory deals with taxation of externalities so that the price of using public goods is equal to the marginal cost of using them. Taxes designed according to this principle are known as Pigouvian Taxes (Pigou (1920)). These are normally set equal to the difference between the private average costs and the private marginal costs so that these two prices are equalized. Pigouvian taxes seem to be very simple when looked upon in isolation but one must remember that all the taxes in the economy makes up the tax system. It is therefore interesting to characterize the optimal tax system when externalities are present and the government have to raise revenue for different purposes (e.g. defence). This approach was taken by Sandmo (Sandmo (1975)) in the standard model for optimal taxation. The result from this was the well known additivity property. A similar result emerges here when externalities are included.

This paper will use the approach introduced by Becker (1965) representing time explicitly in the utility function and used by Kleven (2003) to derive a modified version of the inverse elasticity rule. The purpose of this paper is to extend this result to the situation where externalities are present. The model by Kleven is extended to allow for externalities and the results are compared with those obtained in the initial model.

2. The model

We assume that there are $N + 1$ commodities and H identical households in the economy. Each household is maximizing a utility function given by

$$U_h = U_h(Z_h^0, Z_h^1, ..., Z_h^N) - \phi(\bar{Z}^N), \quad h = 1, ..., H \tag{2.1}$$

where $\bar{Z}^N = \sum_{h=1}^{H} Z_h^N$ is the total consumption of good N in the economy and $Z_h^i = f^i(X_h^i, L_h^i)$, $i = 0, ..., N$ represent the way good Z_h^i is being produced in household h using one market good X_h^i, time L_h^i and production technology f^i. For now we do not impose any restrictions on the production technologies other than saying that every household uses the same technology f^i in the production of Z^i. The function $\phi(\cdot)$ is assumed to be increasing in $\bar{Z}^N$. As a result the total consumption of good Z^N decreases the utility of the households. Assuming that the number of households H is large we make the standard assumption that the individual household behaves as if $\frac{\partial \bar{Z}^N}{\partial Z_h^N} = 0$ which means that the household may know that it affects the total consumption of good Z^N but regards it's contribution as insignificant. Following the approach in Becker (1965) we now write the optimization problem for household h as

$$\begin{array}{cc} \max\limits_{\{X_h^i\}_{i=0}^N \{L_h^i\}_{i=0}^N} & U_h(f_h^0(X_h^0, L_h^0), \ \ f_h^1(X_h^1, L_h^1), \ \ ... \ \ f_h^N(X_h^N, L_h^N)) \\ & \sum_{i=0}^{N} P^i X_h^i \ \ = \ \ wN_h \\ s.t. & \\ & \sum_{i=0}^{N} L_h^i + N_h \ \ = \ \ T \end{array} \tag{2.2}$$

where w is the wage rate, N_h is the amount of time spend on work, P^i it the consumer price of market good X^i and T is the total time available to the consumer.

This description of the household shows that households are not only modelled as consumers but also as producers. Therefore we start by taking a closer look on the production process taking place inside the household. The household seeks to produce the good Z_h^i in the most efficient way. This problem can essentially be seen as an attempt to minimize the production costs of every unit Z_h^i. Letting the factor input coefficients a_{Li} and a_{Xi} be the input of L^i and X^i in the production process and assuming that P^i is fixed the household solves the following problem for every commodity Z^i

$$\begin{array}{ccc} \min\limits_{a_{Xi}, \ a_{Li}} & P^i a_{Xi} + a_{Li} & \\ s.t. & f^i(a_{Xi}, a_{Li}) & = \ \ 1 \end{array} \tag{2.3}$$

Hereby they find the cheapest way to produce one unit of the consumption good Z^i. The solution is characterized by the unit cost functions $a_{Li} = a_{Li}(P^i)$ and $a_{Xi} = a_{Xi}(P^i)$ describing the cost of producing one unit of Z^i measured in factor input. Using the solution to (2.3) and normalizing both the wage rate w and the total time T to 1 we can rewrite

(2.2) as

$$\begin{aligned} \max_{\{Z_h^i\}_{i=0}^N} \quad & U_h(Z_h^0, Z_h^1, ..., Z_h^N) \\ s.t. \quad & \sum_{i=0}^{N} Q^i(P^i) Z_h^i = 1 \end{aligned} \tag{2.4}$$

where $Q^i(P^i) = P^i a_{xi}(P^i) + a_{li}(P^i)$ is the total cost of the consumption good. To see this remember that a_{Xi} and a_{Li} can be written as $a_{Xi} = \frac{X^i}{Z^i}$ and $a_{Li} = \frac{L^i}{Z^i}$. Adding the two constraints in (2.2) and using the normalization of w and T gives the single constraint in (2.4). Note that $P^i a_{xi}$ is the direct cost of consuming X^i and that a_{li} is the value of the time used for consumption which equals the earnings lost due to lower working time. Therefor Q^i is the total cost of consuming one unit of Z^i.

Because this is a standard utility maximization problem we know that the solution will give the factor demand functions $Z_h^i(Q^0(P^0), ..., Q^N(P^N), y)$ and the indirect utility function $V_h(Q^0(P^0), ..., Q^N(P^N), y)$ where y represent artificial non-labour income. We also know that the standard results like Roy's Identity which states that

$$\frac{\partial V_h}{\partial Q^k} = \lambda Z_h^k \tag{2.5}$$

will apply.

Having characterised the consumers behaviour we now focus on the government. We assume that the government seeks to maximize a Bergson-Samuelson type social welfare function

$$W = W(U_1, ..., U_H) \tag{2.6}$$

and that the utility of every household enters additively. Furthermore we assume that the government has a given revenue requirement given by G and that goods 1 to N can be taxed. The governments behaviour can be formulated as

$$\begin{aligned} \max_{\{P^i\}_{i=1}^N} \quad & \sum_{h=1}^{H} (V_h(Q^0(P^0), ..., Q^N(P^N), y) - \phi(\sum_{h=1}^{H} Z_h^N(Q^0(P^0), ..., Q^N(P^N), y))) \\ s.t. \quad & \sum_{i=1}^{N} ((P^i - p^i)(\sum_{h=1}^{H} X_h^i)) = G \end{aligned} \tag{2.7}$$

where p^i is the producer price of commodity X^i and assumed to be constant. This assumption ensures that the tax rate $t^i = P^i - p^i$ is solely determined by the price P^i giving the government full control of the consumer prices through the taxes t_i. It is assumed that good 0 can not be taxed. Having identical consumers we can drop the subscript h and restate the problem as

$$\begin{aligned} \max_{\{P^i\}_{i=1}^N} & \quad HV(Q^0(P^0),...,Q^N(P^N),y) - H\phi(HZ^N) \\ s.t. & \quad \sum_{i=1}^N H(P^i - p^i)a_{Xi}Z^i(Q^0(P^0),...,Q^N(P^N),y) \quad = \quad G \end{aligned} \tag{2.8}$$

The following section will look at the first order conditions for this problem under different assumptions deriving results for the optimal tax structure in these cases.

3. Optimal tax rules

The Lagranian function emerging from (2.8) is given by

$$\begin{aligned} L &= HV(Q^0(P^0),...,Q^N(P^N),y) - H\phi(HZ^N) \\ & -\mu(\sum_{i=1}^N H(P^i - p^i)a_{Xi}Z^i(Q^0(P^0),...,Q^N(P^N),y) - G) \end{aligned} \tag{3.9}$$

and this will form the basis for the following descusions. Assuming that the production technology used by all households are of the Leontief type we restrict the discussion to situations where the coefficients a_{Xk} and a_{Lk} are constant for all k. The general first order condition for (2.8) can now be written as

$$\frac{\lambda - \mu}{\mu} = -H\frac{\partial\phi}{\partial Z^N}\frac{\partial Z^N}{\partial Q^k}\frac{1}{Z^k}\frac{1}{\mu} + \sum_{i=1}^N \frac{\partial Z^i}{\partial Q^k} t_i a_{Xi}\frac{1}{Z^k}, \quad k = 1,...,N \tag{3.10}$$

Assuming that there are no cross price effects in the economy, defining the constant

$$\theta = \frac{\lambda - \mu}{\mu} \tag{3.11}$$

and using Roy's Identity we can simplify (3.10) to

$$\theta = t_k a_{Xk}\frac{\partial Z^k}{\partial Q^k}\frac{1}{Z^k}, \quad k = 1,...,N-1 \tag{3.12}$$

$$\theta = t_N a_{XN}\frac{\partial Z^N}{\partial Q^N}\frac{1}{Z^N} - \frac{H}{\mu}\frac{\partial\phi}{\partial Z^N}\frac{\partial Z^N}{\partial Q^N}\frac{1}{Z^N}, \quad k = N \tag{3.13}$$

giving the optimality condititons

$$\frac{t_k}{P_k} = \frac{\theta}{\alpha_{Xk}\epsilon_k}, \quad k = 1,...,N-1 \tag{3.14}$$

$$\frac{t_k}{P_k} = (\frac{\theta}{\epsilon_N} + \frac{H}{Q^N\mu}\phi^{'})\frac{1}{\alpha_{XN}}, \quad k = N \tag{3.15}$$

where $\alpha_{Xk} = \frac{P_k a_{Xk}}{Q^k}$ is the cost share of X^k in the price of Z^k and $\epsilon_k = \frac{\partial Z^k}{\partial Q^k}\frac{Q^k}{Z^k}$ is the own price elasticity of commodity k. We will now look at some special results emerging from these two conditions.

3.1 The inverse elasticity rule

Assuming that there is no externalities in the model ($\phi^{'} = 0$) and that the cost share for all goods are equal to one ($\alpha_{Xk} = 1$, $k = 0, ..., N$) we get the following characterization of the optimal tax structure

$$\frac{t_k}{P_k} = \frac{\theta}{\epsilon_k}, \quad k = 1, ..., N \tag{3.16}$$

This is precisely the standard result stating that the optimal choice of taxes are inversely related to the own price elasticities of the commodities.

3.2 Modified inverse elasticity rule

To confirm the results found in Kleven (2000) we assume that there are no externalities in the model ($\phi^{'} = 0$). The optimality conditions can then be written as

$$\frac{t_k}{P_k} = \frac{\theta}{\alpha_{Xk}\epsilon_k}, \quad k = 1, ..., N \tag{3.17}$$

which states that the optimal choice of taxes is inversely related not only to the own price elasticity but also inversely related to α_{Xk}. We will now look a little closer on this new parameter. Defining $\alpha_{Lk} = \frac{a_{Lk}}{Q^k}$ as the share of costs related to the time used for consumption of commodity k we see that $1 = \alpha_{Xk} + \alpha_{Lk}$. The condition therefore says that time intensive commodities should carry a higher tax rate than commodities which are less time intensive in the household production. This conclusion is the same as the one found in Kleven (2003) namely that faster modes of transport should be taxed at a lower rate than slower modes.

3.3 The new tax rule

Assuming that there are externalities in the model as well as requiring that it takes some amount of time to consume the commodities we end up with conditions (3.14) and (3.15). The first of these is just the modified inverse elasticity rule. The latter can be characterised as a combination of the modified inverse elasticity rule and a modification of the additivity property found in Sandmo (1975). The conditions state that for goods not generating externalities the modified inverse elasticity rule can be used to find the optimal tax level. For goods generating externalities this rule does not apply directly but the tax should be higher in order to correct for the generated externalities. It is worth recognising that the cost of the externality only affect the tax formula on the good involved in generating the externality.

4. Concluding remarks

The purpose of this paper has been to extend the result obtained in Kleven (2003) to the case where externalities are present. This resulted in a characterization of the optimal tax system which combined the modified inverse elasticity rule and the additivity property found in Sandmo (1975). The new characterization changes the conclusion found in Kleven (2003) that fast modes of transport should carry a lower tax rate than slower modes. The conclusion only holds as long the amount of negative externalities generated by the consumption of the goods are small. If substantial externalities are present the formulas says that we have to increase the taxes in line with the Pigouvian principle.

The results presented in this paper are very preliminary and the conclusions will be clarified and sharpened in the time to come. Other results in the literature will be imbedded in this setup and hopefully some insights will come from this. Insights from earlier work by Samuelson (Samuelson (1954)) dealing with provision of public goods might also be obtained. More specifically the Corlett-Hague result will be considered in the present setup.

References

Auerbach, A. J. and Hines, James R. (2002) Taxation and Economic Efficiency, *Handbook of Public Economics* **3**, Ed. by A. J. Auerbach and M. Feldstein, Elsevier Science B. V.

Becker (1965) A Theory of the Allocation of Time, *The Economic Journal* **75**, 493-517

Corlett, W. J. and Hague, D. C. (1954) Complementarity and the Excess Burden of Taxation, *Review of Economic Studies* **21**, 21-30.

Kleven, H. J. (2000) Optimum Taxation and the Allocation of Time, *EPRU Working Paper Series* **2000-16**, Institute of Economics, University of Copenhagen.

Kleven, H. J. (2003) *Taxation, Time Allocation , and Economic Efficiency*, Ph.d. dissertation, University of Copenhagen, Red Series no. 87-2003.

Pigou, A. C. (1920) *The Economics of Welfare*, Macmillian, London.

Ramsey, F. P. (1927) A Contribution to the Theory of Taxation, *The Economic Journal* **37**, 47-61.

Samuelson, P. A. (1954) The Pure Theory of Public Expenditure, *The Review of Economics and Statistics* **36:4**, 387-389.

Sandmo, A. (1975) Optimal Taxation in the Presence of Externalities, *Swedish Journal of Economics.***77**, 86-97.

Sandmo, A. (1976) Optimal Taxation, An introduction to the literature, *Journal of Public Economics* **6**, 37-54.